CLUSTERING OF FLOATING PARTICLES IN STOCHASTIC VELOCITY WITH SMALL COMPRESSIBLE COMPONENT

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Mots clefs : stochastic velocity, clustering, statistical topography

It is well-known that floating impurities can cluster into narrow, elongated stripes. Existing theories predict the occurrence of clustering in the compressible velocity fields. When we consider a velocity field with potential and solenoidal components, complete clustering is possible only if the potential component magnitude exceeds the solenoidal one [1,2]. However, this result is valid only asymptotically [1,2]. In the ocean, the ageostrophic component of the velocity field is, in most cases, small [3,4]. Thus, for floating particles the potential component of the velocity is also small in comparison with the quasi-geostrophic incompressible component [1]. In the present work, we investigate numerically clustering of floating impurities in a stochastic velocity field with a small potential component. We consider the random velocity in the form

$$\mathbf{U}(\mathbf{R};t)=\gamma\mathbf{U}_p(\mathbf{R};t)+(1-\gamma)\mathbf{U}_s(\mathbf{R};t),\quad (0\le\gamma\le1),$$

where $\mathbf{U}_p(\mathbf{R};t)$ is potential component and $\mathbf{U}_s(\mathbf{R};t)$ is solenoidal one.

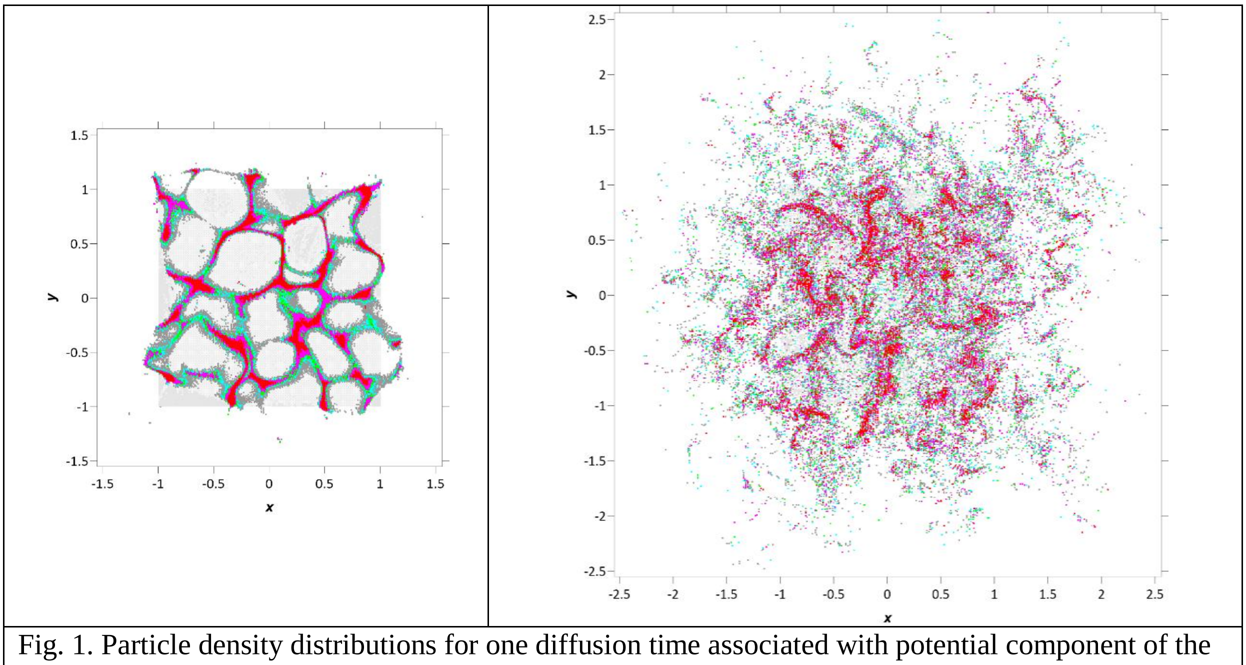

Fig. 1. Particle density distributions for one diffusion time associated with potential component of the velocity. $l=0.16$, $\sigma_U=1.3333$. (left - $\gamma=1$, right - $\gamma=0.2$) The light grey square area is the initial particle distribution.

We show that, in such flows, clustering of the impurities can occur. However, unlike the case with a large potential component, the clustering is much slower process in the case with small potential component. Furthermore, we investigate the dynamics of clustering by means of statistical topography characteristics.

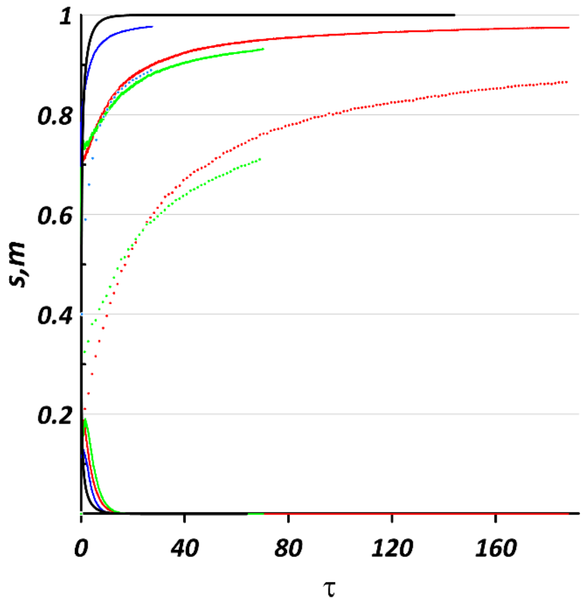

Fig. 2. a single realization of the cluster area and mass for the threshold density value $\bar{\rho}=1$ and for parameters from fig. 1(right).

From the statistical topography point of view in the case of a positive definite density field $\rho(\mathbf{R},t)$, clustering happens in almost every realization (or with probability 1) if the asymptotic relations are simultaneously valid [2]

$$\left\langle s_{\text{hom}}(t,\bar{\rho})\right\rangle \to 0 \ , \ \left\langle m_{\text{hom}}(t,\bar{\rho})\right\rangle \to 0 \text{, when } t \to \infty$$

where $s_{\text{hom}}(t,\bar{\rho}), m_{\text{hom}}(t,\bar{\rho})$ are the specific area and the specific mass of the particles encompassed within the regions of the random field $\rho(\mathbf{R},t)$ exceeding the prescribed level $\bar{\rho}$ [1,5,6]. Fig. 2. shows that the relations are valid for the case $\gamma=0.2$ but are accomplished much slower than in the case without solenoidal component (black line).

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