An HPC implementation of ANM to efficiently compute 3D steady-state bifurcations in incompressible flows

Marc Medale*[1], Bruno Cochelin[2], and Yann Jobic[3]

[1]IUSTI – IUSTI UMR 7343, Laboratoire commun ETiC – France
[2]Laboratoire de Mécanique et d'Acoustique (LMA) – Aix Marseille Univ, CNRS, Centrale Marseille, LMA, Marseille, FRANCE – France
[3]IUSTI, CNRS UMR 7343 – Aix-Marseille Université - AMU – France

Résumé

The present work presents a computationally efficient implementation of a continuation algorithm based on Asymptotic Numerical Method in the framework of large size algebraic systems resulting from the discretization of incompressible Navier-Stokes equations. It is designed to compute branches of solutions for a given range of control parameters, along with the determination of critical values and their corresponding singular solutions. First order predictor-corrector algorithms with pseudo-arc-length parameterization have been widely used for decades [1, 2, 3]. Nevertheless, their step-length adaptivity may be in trouble in the vicinity of bifurcation points leading to a weak computational efficiency and sometimes lack of convergence. The present algorithm stands on high-order predictors produced by the Asymptotic Numerical Method, in which their high-order Taylor series expansion contain valuable information that can be exploited in the course of continuation. Indeed, power series analysis enables to accurately detect and compute simple bifurcation points and to recover optimal step length in the vicinity of bifurcation points [4]. All these features enabled us to perform continuation in 3D incompressible fluid flow inside a sudden expansion channel (expansion ratio E = 3, cross-section aspect ratio 10< =B< =24). We have computed for the first time up to four steady symmetry breaking (pitchfork) bifurcations together with their associated bifurcated branches. The main characteristic of this 3D symmetric expansion configuration is that for a given cross-section aspect ratio the first bifurcation mode induces a top-bottom asymmetry, as in the 2D case, whereas the subsequent ones modulate the former in the span-wise direction with increasing wave numbers [5].

References

H. B. Keller. Numerical solution of bifurcation and nonlinear eigenvalue problems. Academic Press, New York (1977).

E. Doedel, H. B. Keller and J. P. Kernevez. Numerical analysis and control of bifurcation problems (i) bifurcation in finite dimensions. Int. J. Bifurcation and Chaos, Vol. 3, 493–520 (1991).

R. Seydel. Practical bifurcation and stability analysis. Springer-Verlag (1994).

*Intervenant

B. Cochelin and M. Medale. Power series analysis as a major breakthrough to improve the efficiency of Asymptotic Numerical Method in the vicinity of bifurcations. J. Comput. Phys. Vol. 236, 594–607 (2013).

M. Medale and B. Cochelin. High performance computations of steady-state bifurcations in 3D incompressible fluid flows by Asymptotic Numerical Method. Journal of Computational Physics, Vol. 299, pp. 581–596 (2015).

Mots-Clés: 3D steady, state bifurcations, Path, following or continuation methods, Asymptotic Numerical Method, Power series analysis.