

Vertical alignment of two vortices in a continuously stratified quasi-geostrophic flow

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Résumé :

Nous considérons l'interaction entre deux tourbillons quasi-géostrophiques de rapport d'aspect hauteur sur rayon h/r , placés sur deux niveaux verticaux différents. Nous étudions si de telles structures s'alignent naturellement. Dans le cas où les tourbillons occupent des niveaux distincts mais contigus, un tel alignement offre un scénario possible pour expliquer l'accroissement du volume des tourbillons océaniques de moyenne échelle. L'autre mécanisme est la fusion de tourbillons partageant des niveaux verticaux en commun. Nous montrons l'existence d'états d'équilibre penchés où les tourbillons s'alignent obliquement. La plupart des équilibres avec $h/r > 1$ est stable à part dans une région restreinte de l'espace paramétrique. L'instabilité est en général non-destructrice. Les paires de tourbillons aplatis peuvent aussi constituer un équilibre instable lorsque les tourbillons sont modérément séparés horizontalement. Dans ce cas, l'instabilité peut résulter en l'éjection de filaments et en l'alignement des tourbillons.

Abstract :

We consider the interaction between two quasi-geostrophic vortices of height-to-width aspect ratio h/r , lying at two different vertical levels. We investigate whether such structures naturally align. In the case vortices occupy distinct yet contiguous levels, alignment offers a possible scenario to explain the growth in volume of oceanic meso-scale vortices. The other growth mechanism is the merger of vortices sharing common vertical levels. We show that there exist tilted equilibrium states where vortices align slantwise. Most equilibria for prolate vortices ($h/r > 1$) are stable apart in a very narrow region of the parameter space. The instability is however normally non-destructive. Pairs of oblate vortices may nonetheless be in an unstable equilibrium when moderately offset horizontally. In this case, the instability may result in the shedding of filamentary potentially vorticity from the vortices and result in their further alignment.

Mots clefs : Quasi-geostrophy, Vortex interactions

1 Introduction

Meso-scale oceanic vortices contain a large part of the oceans' kinetic energy, and they contribute to at least 50 % of the mass transport in the oceans [1]. In this paper, we revisit the problem of vortex alignment

in a continuously-stratified quasi-geostrophic flow. The vertical alignment of two vortices is one of the mechanisms put forward to explain, in physical space, the growth in volume of such high-energy oceanic vortices. The problem has already been explored in of a two-layer model by Polvani (1991) [2] and in continuously stratified fluid by Viera (1995) [3], Sutyrin *et al.* (1998) [3] and Reasor and Montgomery (2001) [5]. In this work, we consider two disjoint vortices of mean height-to-width aspect ratios h/r , rather than a distorted columnar vortex. Vortices are separated in the vertical direction by a layer of fluid of thickness Δz . We consider the cases $\Delta z = 0$ and $\Delta z = 2h/61$, $6h/61$, and $12h/61$, where h is the half-height of the vortices.

We first determine families of equilibrium states (V-states) using two different approaches. In each family the vortices have a specified aspect ratio h/r , and the vortices are separated by a specified vertical offset Δz . Equilibria within the family differ by the horizontal distance ℓ_x separating their centre. We study the linear stability of the equilibria, to show that most equilibria are stable, in particular if $h/r > 1$. Finally we explore the nonlinear evolution of both unstable equilibria and of pairs of upright-standing vortices which are not initially in mutual equilibrium. We show that, in the latter case, the vortices tilt to approach the nearby equilibrium.

2 Mathematical model

We consider a rapidly rotating, continuously stratified fluid. We denote the Rossby number $Ro = U/(fL)$, and the Froude number $Fr = U/(NH)$. Here, U is a typical horizontal velocity scale, L and H are typical horizontal and vertical length scales respectively, f is the Coriolis frequency while N is the buoyancy frequency. For the sake of simplicity both f and N are assumed constant. If $Fr^2 \ll Ro \ll 1$, the flow is accurately modelled by the quasi-geostrophic equations

$$\frac{Dq}{Dt} = 0,$$

$$q = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2},$$

where q is the materially conserved quasi-geostrophic potential vorticity anomaly, φ is the streamfunction from which the layerwise two-dimensional, geostrophic, advecting velocity $\mathbf{u} = (u, v, 0)$ derives

$$u = -\frac{\partial \varphi}{\partial y}, \quad v = \frac{\partial \varphi}{\partial x},$$

and finally z is the coordinate along the vertical direction after multiplication by the constant factor N/f .

We consider vortices consisting of uniform potential vorticity anomaly q which are fully described by the boundary (a surface with an infinite number of degrees of freedom). We also make use of a simplified dynamical model, the Ellipsoidal Model (ELM) derived by Dritschel *et al.* (2004) [6] which represents vortices by fluid ellipsoids and filters out non-ellipsoidal deformations. In this model each vortex has been only 5 degrees of freedom. Vortex i is defined by its centroid position $\mathbf{X}_i = (X_i, Y_i, Z_i)$ and a symmetric matrix \mathcal{B}_i such that the boundary of the ellipsoid is defined by $(\mathbf{x} - \mathbf{X}_i)\mathcal{B}_i^{-1}(\mathbf{x} - \mathbf{X}_i)^T = 1$, where $\mathbf{x} = (x, y, z)$ is the line position vector. Note that the lack of vertical advection means that Z_i and $(\mathcal{B}_i)_{3,3}$ are time-independent. The time evolution of X_i and \mathcal{B}_i are governed by a Hamiltonian system where the Hamiltonian is the total energy $\mathcal{H} \propto \iiint q \varphi dV$ [6].

3 Results and discussion

We determine pairs of vortices in mutual equilibrium. In the first approach, vortices are modelled by ellipsoids and non-ellipsoidal deformations are filtered out. We also determine numerically equilibrium states for the full quasi-geostrophic dynamics, which includes non-ellipsoidal deformations. In both approaches the equilibria vortices are tilted towards each other as seen in figure 1.

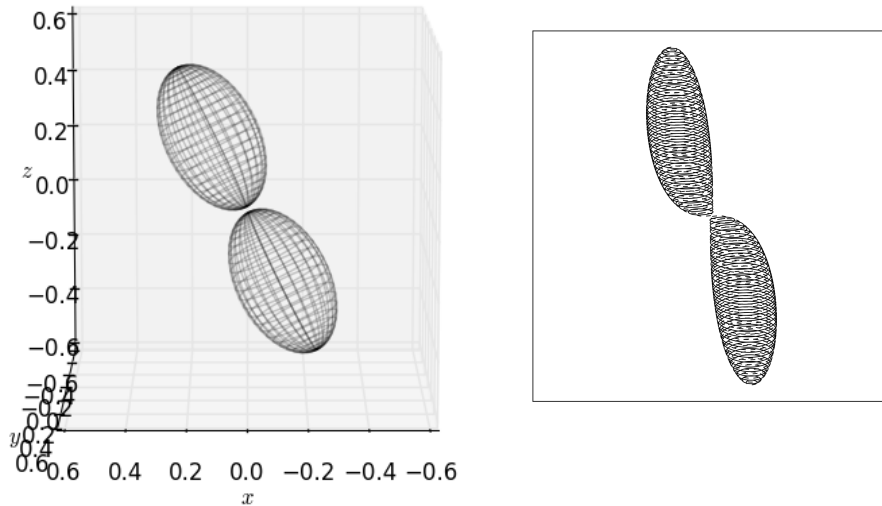


FIGURE 1 – Examples of equilibria for $h/r = 1.5$ in the narrow range where the vortex pair is unstable. Ellipsoidal equilibria (left) and full QG equilibria (right).

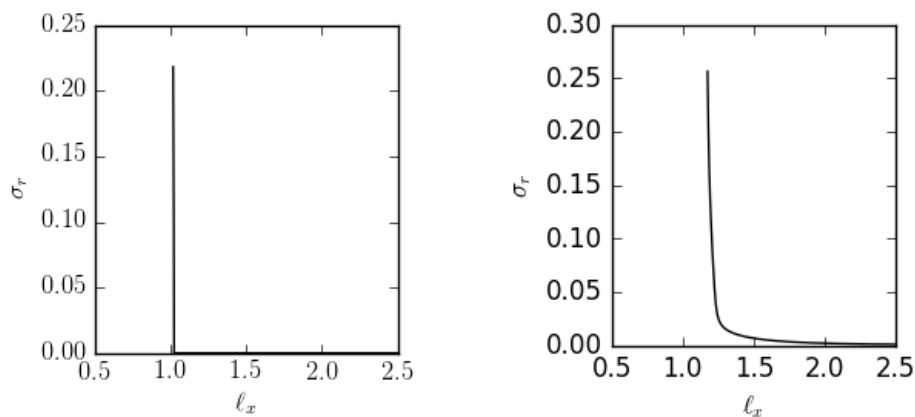


FIGURE 2 – Maximum growth rate $h/r = 0.5$ vs the horizontal centroid separation l_x for $\Delta z = 0$. Ellipsoidal equilibria (left) and full QG equilibria (right).

We found a good agreement between the two approaches over wide parameter space. Figure 2 shows the maximum growth rate for $h/r = 0.5$ and $\Delta z = 0$ as a function of the horizontal centroid separation l_x for moderate values of l_x .

Most of the equilibria are stable apart for a very narrow region of instability if the vortices are prolate. In this case the instability is not destructive, and the instability leads to small amplitude pulsation of the vortices.

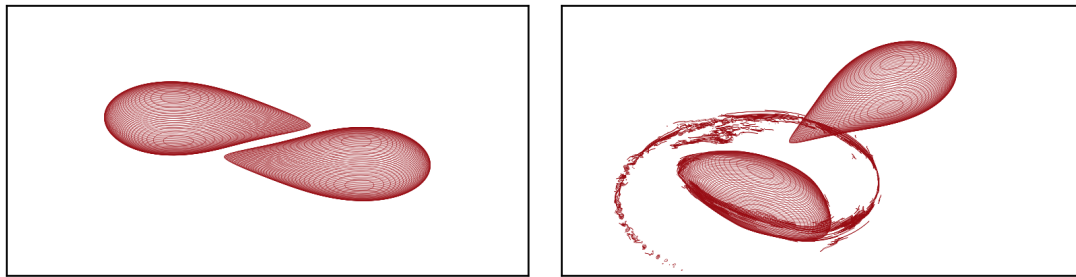


FIGURE 3 – Example of nonlinear evolution of two vortices in unstable equilibrium with $h/r = 0.25$, $\Delta z = 0$ (end of the branch of equilibria). Vortices are viewed at an angle of 65° from the vertical direction.

On the other hand, oblate vortices may be unstable when the vortices are moderately offset in the horizontal direction, and typically when the vortices are partially vertically aligned, see for example figure 3.

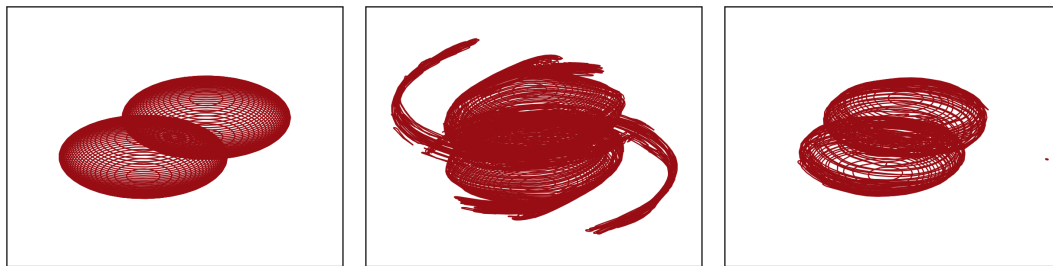


FIGURE 4 – Example of nonlinear evolution of two non-equilibrium, spheroidal vortices $h/r = 0.25$, $\Delta z = 0$ (end of the branch of equilibria). Vortices are viewed at an angle of 65° from the vertical direction.

When starting from non-equilibrium, upright-standing, spheroidal vortices, the vortices tend to tilt to reach the nearby titled equilibrium. If the vortices are prolate, the nearby equilibrium is generally stable and the transition from the upright-standing position a near titled equilibrium occurs with very little filamentation. Filamentation occurs near the vertical tips of the vortices as waves propagate up and down the vortex boundaries. If the vortices are oblate, the nearby equilibrium may be unstable, and the non-linear evolution of the up-right standing vortices is more complex as shown in figure 4. More significant filamentation may occur allowing the vortices to further align while conserving the integral invariants in the flow.

Références

- [1] Zhang, Z., Wang, W. and Qiu, B., 2014, Oceanic mass transport by mesoscale eddies. *Science*, **345**, 322-324.
- [2] Polvani, L.M., 1991, *J. Fluid Mech.*, **225**, 241-270.
- [3] Viera, F., 1995, *J. Fluid Mech.*, **289**, 29-50.
- [4] Sutyrin, G.G., McWilliams, J.C. and Saranavan, R., 1998, *J. Fluid Mech.* **357**, 321-349.

- [5] Reasor, P.D. and Montgomery, M.T., 2001, *J. Atmos. Sci.*, **58**, 2306-2330.
- [6] Dritschel, D.G., Reinaud, J.N, and McKiver. W.J., 2004, *J. Fluid Mech.*, **505**, 201-223.