Analytical Solution to the Stability of Gravity-driven Stratified Flow of Two Liquids over an Inclined Plane

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Résumé

This paper presents a theoretical study of the stability of a gravity-driven stratified flow of two liquids with an open surface. The flow is controlled by three non-dimensional parameters, say the viscosity ratio , the density ratio and the thickness ratio . In industry, the fluids handled are polymer melts and different defaults are often observed on the final solid products. These defaults are the results of instabilities occurring in the previous flow. The instabilities occurring in this flow system were considered by Hu et al. [1] focusing on the effect of density stratification in both configurations : without and with inertia. The inertialess configuration with viscosity stratification was studied by Kao [2, 3], Loewenherz and Lawrence [4]. Millet et al. [5] achieved a numerical study of the linear stability of a two-layer non-Newtonian fluids obeying the four-parameter Carreau inelastic model with the three control parameters released. Notably, they affirm that an analytical study is possible in the Newtonian case. The aim of this paper is to perform such analytical study. First of all, the basic laminar steady flow was considered. The effect of each of these control parameters on the velocity in each layer, on the flow rate in each layer, as well as on the total flow rate were derived from that analytical solution. Then, the spatiotemporal deformation of a given cross-section vs. the control parameters was achieved. Also, the respective effects of the control parameters on shear stress distribution was determined. For any values chosen for the different control parameters, the graphs revealed a concentration of shear stress at the interface between both liquids and at the free surface, as well. Therefore, these two locii are the seats for the occurrence and the development of interfacial and surface instabilities and the effect of each control parameter on the shear stress has been quantified.

The temporal stability of the flow system was then investigated with respect to long waves. From the Navier-Stokes and continuity equations associated with appropriate boundary conditions, an Orr-Sommerfeld equation was built and solved by a perturbation method for long waves. The analytical expression of the wavespeed was stated and the effect of the different control parameters on wavespeed and on ratio of the amplitudes of the interface and interface. Finally it was found that shear waves are damped while while the general marginal

*Intervenant

stability curve was derived from the asymptotic solutions for long waves and short waves.

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Mots-Clés: Analytical solution, Density, Free surface, Gravity current, Inclined plane, Interface, Layer thickness, Stability, Stratified flow, Viscosity