Comparative time variant reliability study of a T-bonded structure.

M.MEJRI[a], A. BOUREHLA[a]

a. Unité SACES 17DN01, École de l'aviation Borj El Amri, Tunis, Tunisie
mejri.eaba@gmail.com

Résumé :

Ce travail présente une étude mécano fiabiliste d'une structure collée en T. Cette structure, largement utilisée dans les assemblages (dans divers domaines d'utilisation et à différentes échelles), peut avoir des propriétés variables et peut évoluer dans des milieux sévères. Elle peut aussi subir des chargements aléatoires et stochastiques. Afin de prédire la réponse de telles structures dans le temps il faut prendre en considération ces différents aspects (entre autres). Pour ce travail deux cas sont présentés, le premier s'intéresse à l'étude de la fiabilité et au calcul des probabilités de défaillance instantanée et cumulée d'une structure en T modélisée avec la théorie RDM. La seconde partie traite du cas non linéaire modélisé avec la théorie des éléments finis. Pour les deux cas deux méthodes différentes de calcul de probabilité de défaillance cumulée sont comparées. Les joints de colle sont modélisés avec des éléments cohésifs afin de réduire les coûts numériques de l'approche.

Abstract:

This work introduces a time variant reliability approach based on the estimation of the durability of a T-bonded structure. This widely used structure (construction, naval field, aeronautical field...) can have various material properties, it can also evolve in an environment characterised by severe conditions. Loads can also be time dependent and can have random character. If we want to model correctly the structural response all these aspects must be taken into account. For this work, two studies will be introduced. The first part is based on an analytical elastic study of the T. For the second part of the work cohesive zone elements are used to model the adhesive in order to simplify the numerical study of the adhesively bonded assembly and limit the numerical cost. The cumulative probability of failure is computed and two methods are compared. An example showing the differences between the two methods is presented in the sequel

Keywords: Adhesively bonded structure, Mechanics, FEM, Computational Mechanics, Reliability

1 Introduction

The manner in which a mechanical structure responds to loading depends on the type and magnitude of the applied loads, on the random character of material properties and on structural stiffness. In practice, some design parameters and their evolution in time are not completely mastered. For instance, in the case of assembling elements of composite structures or metallic materials by adhesive bonded joints, variability in joint thickness or presence of possible defects can't be ignored. Thus, in order to improve the service life, those phenomena must be taken into account. Whether the response is considered satisfactory depends on the requirements which must be satisfied. These include safety of the structure against collapse, damage, deflection ... As industrial applications are concerned; the structural response is, in general, non-linear. So a non-linear finite element analysis must be performed to model this response if elastic or linear model is insufficient. The use of non-linear finite element analysis is nowadays widely recognized as the most effective method to assess the response of industrial-type structures. Its combination with time-variant reliability methods is thus a way to optimize their maintainability

To model this kind of issues, strongly non-linear time-dependent problems have to be solved, leading to important computational costs. To limit numerical costs, finite element model has to be optimized. Thus, cohesive zone models are used in this study.

2 Time variant reliability approach

The assessment of risks stemming from the introduced phenomena is an essential issue in predicting the service life. Time-independent reliability does not provide sufficiently relevant and realistic information. In fact loads and material properties can evolve in time. It is thus necessary to use the theory of time-dependent reliability to take into account the time dependence of the phenomena and the loading. The coupling of the structural reliability software evaluation with a non-linear finite element analysis requires robust mechanical models to ensure the quality of durability of a structure in time [1,2].

The classical approach for time-variant reliability analysis relies on the computation of out-crossing rates (of the limit-state surface). Among the existing methods, the PHI2 method is interesting as it allows the use of time-independent reliability methods such as FORM (First-Order Reliability Method).

The PHI2 method considers the assessment of the probability as a two component parallel system analysis. If FORM is used, two analyses give the coordinates of the tangent hyper-plane to the limit-state surface and the classical reliability products such as the reliability index β at times t and $t + \Delta\tau$ (Figure 1). They are lead by the Abdo-Rackwitz algorithm associated with a Newton-Raphson line search. The correlation between the two events $A = \{G(t, \mathbf{X}(t,\omega)) > 0\}$[1] (safe domain) and $B = \{G(t+\Delta\tau,\mathbf{X}(t+\Delta\tau,\omega)) \leq 0\}$ (failure domain) is noted ρ_{GG}.

$$\text{Where } \rho_{GG}(t, t+\Delta\tau) = -\alpha(t).\alpha(t+\Delta\tau)$$

A time-invariant reliability analysis corresponds to assess:

$$P_{f,i}(T) = prob(G(T, \mathbf{X}(T,\omega)) \leq 0) \; .$$

This instantaneous probability of failure $P_{f,i}$ differs from the cumulative probability of failure $P_{f,c}$, which corresponds to the following assessment:

$$P_{f,c}(0,T) = prob(\exists\, \tau \in [0,T], \text{such as } G(\tau, \mathbf{X}(\tau,\omega)) \leq 0).$$

[1] $\mathbf{X}(t,\omega)$ denote the set of random variables used in the mechanical problem, t being the studied time and ω standing for the outcome in the space of outcomes $\mathbf{\Omega}$

$P_{f,c}$ is assumed to be defined with respect to the probability of first out-crossing. Thus, when the limit-state function G decreases on [0,T], then: $P_{f,c}(0,\tau) = P_{f,i}(\tau) \quad \forall \tau \leq T$. In other cases, a different approach has to be considered. The most common one relies on the computation of the out-crossing rate which can be defined by:

$$\nu^{+}(t) = \lim_{\Delta\tau \to 0, \Delta\tau > 0} \frac{prob(A \cap B)}{\Delta\tau} \quad \text{Where} \quad \begin{cases} A = \{G(t, \mathbf{X}(t,\omega)) > 0\} \\ B = \{G(t+\Delta\tau, \mathbf{X}(t+\Delta\tau,\omega)) \leq 0\} \end{cases}$$

The mean number of out-crossings, corresponding to the integral on the time interval of the out-crossing rate, gives the upper bound of $P_{f,c}$:

$$\max_{0 \leq t \leq T}\left[P_{f,i}(t)\right] \leq P_{f,c}(0,T) \leq P_{f,i}(0) + \int_0^T \nu^{+}(t)\, dt$$

Introducing the repartition of the binormal law Φ_2, the first order evaluation of the out-crossing rate by the PHI2 method follows:

$$\nu^{+}_{PHI2}(t) = \frac{\Phi_2\left(\beta(t), -\beta(t+\Delta\tau), \rho_{GG}(t, t+\Delta\tau)\right)}{\Delta\tau}$$

In this expression, the choice of $\Delta\tau$ is crucial; discussions on it are developed in [3]. To facilitate this choice, the method has been improved in [4] by reconsidering the formulation by introducing the following quantity:

$$f_t(\Delta\tau) = \text{Prob}\left(\{G(t, \mathbf{X}(t,\omega)) > 0\} \cap \{G(t+\Delta\tau, \mathbf{X}(t+\Delta\tau,\omega)) \leq 0\}\right)$$

Since events are apart, $f_t(0) = 0$.

$$\nu^{+,Sudret}_{Phi2}(t) = \left\|\underline{\acute{\alpha}}(t)\right\| \Phi(\beta(t)) \Psi\left(\frac{\acute{\beta}(t)}{\left\|\underline{\acute{\alpha}}(t)\right\|}\right) with \;\; \Psi(t) = \varphi(t) - t\,\Phi(-t) \;\; \text{and}$$

Within this equation, the notation " ' " is used to denote the derivative of function with respect to time, evaluated trough finite differences. φ and Φ represents respectively the normal law density and repartition functions. Both expressions of the out-crossing rate are compared in the sequel.

Figure 2 shows the implementation scheme of the direct combination of time-variant reliability methods with finite element analysis.

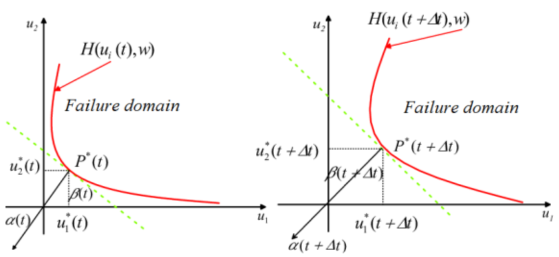


Figure 1: Evolution of the reliability products during Δτ.

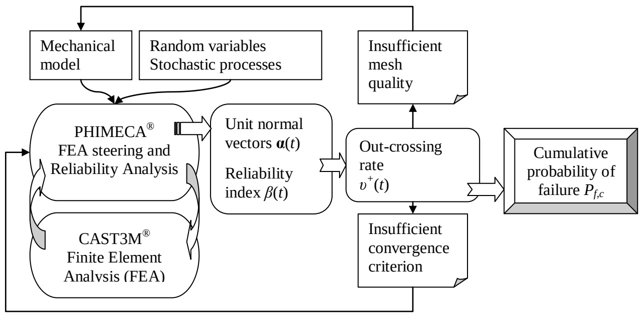


Fig. 2 Implementation scheme of the combination

In this paper, the reliability analysis is performed by the software PHIMECA® which enables the combination with the finite element software CAST3M®.

3 Mechanical model

If adhesive plane layer ensures the junction between two bodies, it can be modeled with interface elements placed between solid bodies. This modeling is based on Dugdal-Barenblatt cohesive zone approach and is justified by the fact that the thickness of the cohesive zone is negligible compared to the assembly dimensions. Thus, adhesive layer can be modeled with a zero thickness zone dividing the structure into two parts. Moreover, in order to model delamination and crack propagation at the interface a damage model can be introduced. This choice implies the limitation of the possible zone of crack advance which becomes limited to the zero thickness zones. It also allows the mesh dependence to be limited for the study of the crack propagation in the adhesive [5, 6].
This approach allows taking into account the effect of adhesive degradation particularly close to defects.
A parametric Valoroso & al model [7] is used to describe the interface elements behavior. Damage is assumed to occur when the joint works in traction or shear and to be constant if solicitation became compressive. For the sequel, the following notation are adopted: D represents the damage, K_n^+, K_s respectively the normal and tangential damaged stiffness and K_n^- is the compressive undamaged interface stiffness.
The constitutive equation of the model can be summarized as:

$$\Psi([U],D) = \frac{1}{2}(1-D)[K_n^+\langle[U_n]\rangle_+^2 + K_s\langle[U_s]\rangle^2] + \frac{1}{2}K_n^-\langle[U_n]\rangle_-^2$$

$$Y_m = -\frac{\partial\psi}{\partial D} = \frac{1}{2}K_n^+\langle[U_n]\rangle_+^2 + \frac{1}{2}K_s[U_s]^2 = \frac{1}{2}K_n^+\delta^2 = \frac{1}{2}K_n^+(\langle[U_n]\rangle_+^2 + \alpha^2[U_s]^2)$$

Y_m is the thermodynamic force and Ψ is the energy of the interface.
This model taking into accounts both normal and shear stiffness evolution will be used in the sequel to model the interface especially the degradation of the adhesive.

4 T-bonded structure under mixed load

The aim of this work is to study the structural response of a widely used structure the T. This structure is described in the figure below.
The first part of this work consists on studying the structural response in elasticity and in testing the feasibility of the time variant reliability approach. The interest of such approach is to evaluate the sensitivity of the reliability model to design parameters.
The second part of the study consists on modeling numerically the T and on evaluating the impact of the degradation on the computation of the probability of failure.
Figure 2 presents the studied T structure.

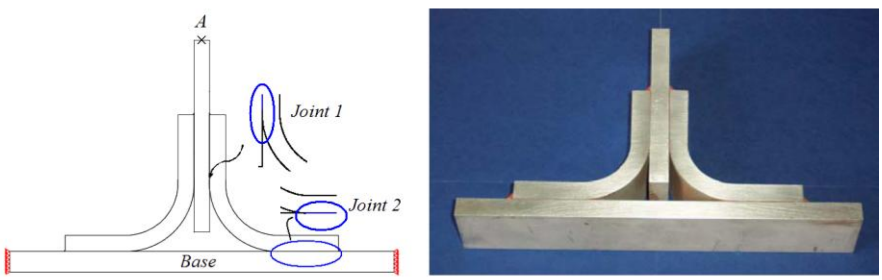


Fig. 2 Studied structure

4.1 Elastic study (approach based on analytical model)

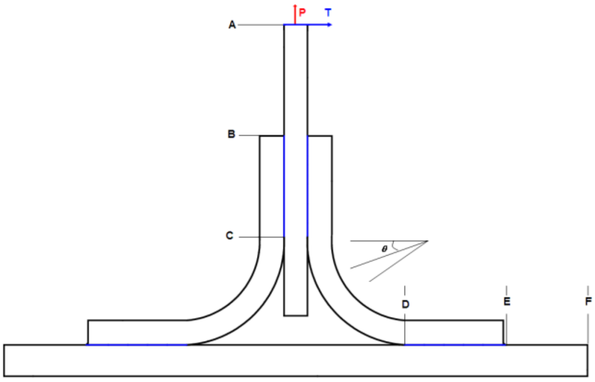


Fig. 3 Geometric parameters of the structure

It was demonstrated Broughton and al that the stiffness of the T can be written as:

For normal stiffness: $K_n = \frac{P}{\delta_n} = \frac{E_a}{6}\left[\frac{\left({b_2}^2+b_2 b_3\right)^3}{{b_2}^3\left(R^2 L_{DE}+R{L_{DE}}^2+\frac{{L_{DE}}^3}{3}\right)+\left(\frac{3\pi}{4}-2\right)(b_2+b_3)^3}\right]$

For tangential stiffness:

$$K_t = \frac{T}{\delta_t} = \frac{E_a}{6}\left[\frac{3{b_1}^3(b_2+0.5b_1)^3{b_2}^3}{{L_{AB}}^3(b_2+0.5b_1)^3{b_2}^3+{L_{BC}}^3(b_1)^3{b_2}^3+\frac{3\pi}{4}R^3(b_2+0.5b_1)^3{b_1}^3}\right]$$

The design function is based on the maximum value of the total deflection ($\delta_t \in s\square ear, \delta_N \in traction$). So the structure is considered to be unsafe if the deflection is higher than a specific value. This case of study was tested with the set of parameters introduced in Table1. The obtained results can be summarized in the figure 4.

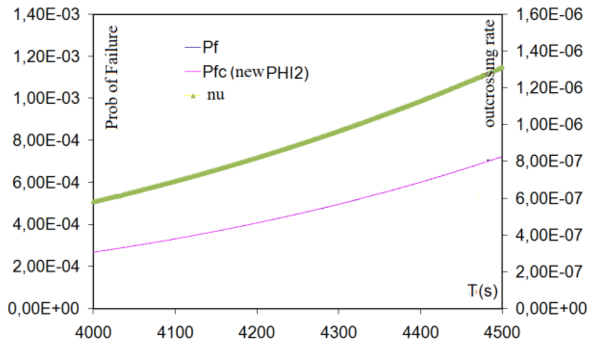


Fig. 4 Comparative probability of failure (elastic case)

Variable	b_2	b_3	L_{DE}	P	R
Sensitivity : α_i	0.5585 (+)	0.5585 (+)	-0.5408 (-)	-0.2492 (-)	-0.1468 (-)
Sensitivity : α^2_i	31.19%	31.19%	29.25%	6.21%	2.16%

Table 1. Sensitivity of the elastic model

The results show the feasibility of the study. They also show the perfect correlation between the two variant of the Phi2 method in the elastic case. Other cases of study (non-introduced in this paper) show that the model is highly sensitive to the angle formed by the traction and the shear directions[2]. The criterion of failure based on the total deflection is reversible (when loading stops, structure returns to its first configuration), the computation of a cumulative probability of failure based on such criterion must be reconsidered. Moreover, it's important to mention that the elastic parametric study is unable to take into account the impact of damage or cracking during the test. To overcome these aspect, non linear finite element based approach is used in the sequel

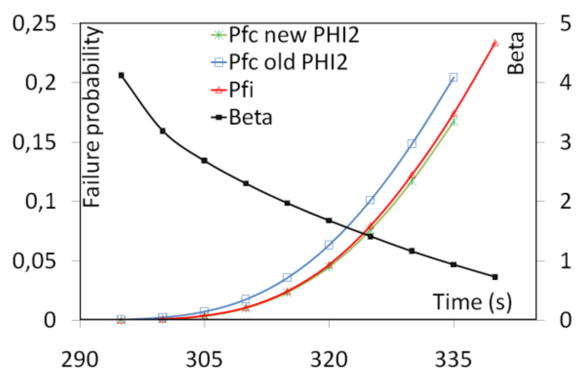
4.2 Finite element model and results

The same case of T structure under traction load is treated in this paragraph. As the problem is symmetric, only one half of the structure is considered. The feasibility of this kind of study was shown in [6] .The studied structure (figure 2) contains two joints; the degradation of each one of the adhesive joint has a specific impact on the total stiffness evolution. Thus a global criterion taking into account this aspect has to be written. To solve this problem, the structure is considered as a parallel system. The failure of one component implies the failure of the whole system. For F.E. simulation, the same materials properties are retained (E = 70000 MPa, ν = 0.34, K_n = 6350 N/mm^3, G_I = 0.47 J/mm²) and each joint is modeled with a 200 cohesive zone elements. The selected failure criterion can be written as follows:

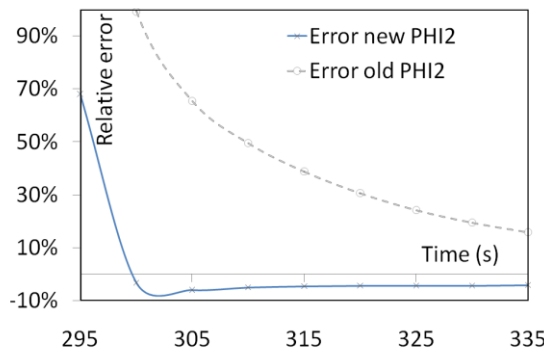
$$G(t) = D_{\text{Limit}} - D_{\max}, \text{ with } D_{\max} = \max_{i=1,2}\left(\int_{\text{joint i}} \delta(x)\, D(x)\text{d}x \Big/ \int_{\text{joint i}} \text{d}x\right)$$

Variable	Distribution	Mean	St-dev	Truncation
D_{Limit}	Normal	5E-2	10%	4E-2..6E-2
G_I(J/mm^2)	Normal	0.51	10%	0.45..0.57
K_n (N/mm^3)	Normal	6350	10%	6000..6700
D_y (mm)	Normal	0.3	10%	0.27..0.33

Table 2. Random variables distributions



(a)Failure probability



(b) Error estimation

Fig. 5 Failure probability computation and errors estimation

Figure 5 presents the evolution of the failure probability (instantaneous and cumulative) in time and the difference between the two methods errors. It shows also that for a finite element based study, the

[2] For this paper only results taking into account normal loading are introduced.

new Phi2 method allows significant reduction of relative errors (between cumulative and instantaneous probability of failure). The relative error for the second variant of the Phi2 method decreases faster than the old Phi2.

As expected, the new PHI2 method ensures a better accuracy results compared to old method results. One can note that damage limit value D_{Limit} is quiet low; this value is chosen to ensure fast F.E computations and do not affect the convergence scheme. We also note that the used reliability criterion, based on the computation of the maximal value of damage, gives approximately the same results if compared with the results of a parallel system analysis (the total probability of failure is supposed to be the sum of the probabilities of failure of the different components of the system). For the selected example we have noted that the maximal value is obtained always for the same adhesive joint (bottom joint), failure probability for the second joint is very low (E-5 order). As a consequence, the total failure probability can be supposed to be the failure of the most damaged joint this result is in agreement with recent results [8].

5 Conclusions

The impact of random parameters on the life time of a T bonded structure is studied. First an elastic case is introduced; this example shows the feasibility of the approach and the accuracy of the new Phi2 method. Then numerical results for the combination of time-variant reliability methods and the non-linear finite element simulation of the degradation of an adhesive in an assembly are presented. The necessary effort to compute the cumulative probability of failure in terms of accuracy and computational cost are highlighted. For this example, instable crack propagation can appear. Specific technique using local control methods (arc length type control) have to be used to ensure the convergence of the computations.

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