Viscous shear effect in non-Newtonian lubrication of finite porous elastic bearings

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Abstract:

In this study, viscous effects on finite porous flexible journal bearings performances are introduced by using Darcy-Brinkman model. Based on Stokes micro-continuum mechanics, and considering bearing elastic deformation determined by Winkler model, an elastohydrodynamic modified Reynolds equation type is derived. The porous bearing flow is described by the complete Darcy-Brinkman model including viscous shearing forces. The film and porous governing equations are coupled at the porous elastic interface by the continuity of pressure, velocities and viscous shear stresses. As this fluid-porous bearing interaction problem necessitates numerical solution, the equations are discretized employing finite differences, and the successive over-relaxation scheme is used to solve iteratively the obtained algebraic equations. The fixed-point technique is advocated to solve sequentially this coupled problem. To show the viscous shear effects, bearing characteristics including load capacity and friction factor are compared to those obtained from Darcy model with Beavers-Joseph slip conditions, and discussed for different couple stress values, permeabilities and elastic deformation parameters.

Keywords: Finite journal bearing; Non-Newtonian lubricants; Porous elastic bearings; Viscous shear effects; Numerical study.

1 Introduction

Porous journal bearings (PJBs) have the advantage of reducing the need for certain lubricating equipment (oil pipes, pumps, etc.) as well as reducing other problems related to lubrication mechanism. One of the advantageous features of the porous bearings is that, no external supply of lubricant is required for running-in. It was Morgan et *al.* [1] who first presented an analytical study of porous bearings lubricated with Newtonian fluid. The requirement of the high performances and long-life bearings in industrial applications have increased the use of non-Newtonian lubricants. Only few contributions have been presented on PJBs lubricated by non-Newtonian couple stresses fluid [2-3]. Recently, Sakim et *al.* [4] presented a primary study on finite flexible PJBs using Darcy model,and considering Beaver-Joseph velocity conditions at the film-porous interface (SFM). According to their results, the deformation effects influence the porous bearing characteristics especially when they operate at high eccentricities. From the above studies, it was predicted that couple stresses lubricants improve the porous bearing characteristics. However, we have no idea how the viscous shear forces of the Darcy-Brinkman model (BM) affect finite porous elastic journal bearings lubricated by non-Newtonian couple stresses fluid.

The present article introduces the viscous shear forces effects in couple stresses lubrication of finite porous elastic journal bearings. Making use of the Stokes micro-continuum mechanics to model non-Newtonian effect within the fluid film and considering bearing elastic deformation determined by Winkler model [5], an elastohydrodynamic (EHD) modified Reynolds equation type is derived. The porous bearing flow is described by the complete BM including viscous shearing forces. To show the viscous shear forces effects, bearing characteristics including load capacity and friction factor are compared to those obtained from SFM, and discussed for different couple stress values, permeabilities and elastic deformation parameters.

2 Problem definition and governing equations

The coordinate system and the geometrical configuration of finite porous elastic journal bearing are shown on Figure 1. It is assumed that the journal and porous bearing are circular and their surfaces are smooth, and the load W is applied in vertical direction. The journal of radius R is rotating with a uniform tangential velocity U about its axis within a porous bearing of initial thickness H_0, permeability k, Young's modulus E and of Poisson's ratio ν. The bearing and the journal are supposed to be aligned, and separated by a non-Newtonian lubricant fluid film of thickness h.

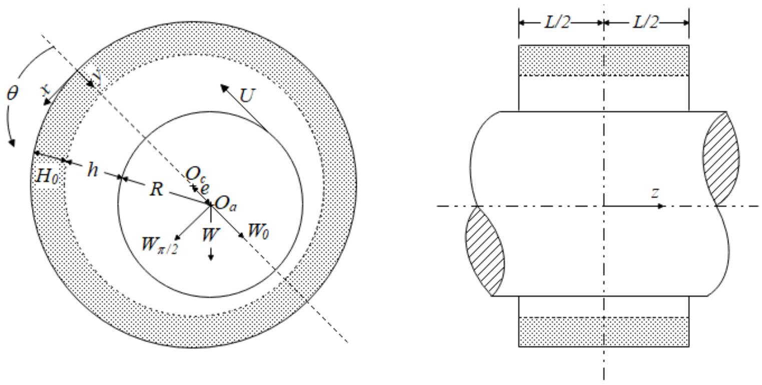

Figure 1: Schematic diagram of porous journal bearing

2.1 Film equations

In the film fluid, using the usual assumption of hydrodynamic lubrication, the continuity and motion equations are written in Cartesian coordinates in the case of an incompressible, laminar, isothermal flow for couple stress [6] fluid in the absence of body forces and body couples:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-l^2\frac{\partial^4 u}{\partial y^4} \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial z}=\frac{\partial^2 w}{\partial y^2}-l^2\frac{\partial^4 w}{\partial y^4} \tag{4}$$

where u, v, w, μ and p are respectively components of the lubricant in the x, y, and z directions, the dynamic viscosity and the pressure in the film. $l=\sqrt{\eta/\mu}$ is the couple stress parameter, where η represents a material constant responsible for the couple stress fluid property.

The boundary conditions to arrive at the velocities of the lubricant are: continuity of the velocity field at the porous elastic interface, vanishing of couple stresses on the journal surface and porous elastic interface, and no-slip boundary conditions at the journal surface.

Integrating the continuity equation (1) with respect to y from H to $H+h$ and using analytical solutions u and w of motions equations (2) and (4), accounting for the corresponding boundary conditions, one can obtain the EHD modified Reynolds equation in the film fluid:

$$\begin{aligned}\frac{\partial}{\partial x}\left(f(h,l)\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(f(h,l)\frac{\partial p}{\partial z}\right)&=\mu v^*_{y=H}+\mu\frac{\partial\left(\frac{h}{2}\left(U+u^*_{y=H}\right)\right)}{\partial x}\\&+\mu\left(u^*_{y=H}-U\right)\frac{\partial H}{\partial x}+\mu\frac{\partial\left(\frac{h\,w^*_{y=H}}{2}\right)}{\partial z}+\mu\,w^*_{y=H}\frac{\partial H}{\partial z}\end{aligned} \tag{5}$$

where $f(h,l)=\frac{h^3}{12}+2l^3\tanh\left(\frac{h}{2l}\right)-hl^2$

Considering the bearing deformation under hydrodynamic pressure, the film and porous thicknesses, are written as follows:

$$h(x,z)=C(1+\varepsilon\cos\theta)+\delta \tag{6}$$

$$H(x,z)=H_0-\delta \tag{7}$$

where C is the radial clearance, $\varepsilon=\frac{e}{C}$ is the journal eccentricity ratio and δ is bearing deformation calculated employing Winkler model [5]:

$$\delta=\frac{H_0(1+\nu)(1-2\nu)}{E(1-\nu)}p(x,z) \tag{8}$$

where ν denote the Poisson's ratio, E is modulus of elasticity, respectively.

2.2 Porous bearing equations

The film additives are assumed of sufficient size not to infiltrate ingoing the porous elastic bearing. The porous elastic bearing is homogeneous, isotropic and saturated by an incompressible Newtonian fluid having an identical shear viscosity like that of the film. Since the bearing thickness is thick, the system of Cartesian coordinates is not adequate for describing the porous flow and, therefore, Cylindrical coordinates are used [7]. The porous flow is described by Darcy-Brinkman model:

$$\left(\frac{\mu}{k}+\frac{\mu^*}{r^2}\right)v^*=\mu^*\left(\frac{1}{r}\frac{\partial v^*}{\partial r}+\frac{\partial^2 v^*}{\partial r^2}+\frac{\partial^2 v^*}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2 v^*}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u^*}{\partial \theta}\right)-\frac{\partial p^*}{\partial r} \tag{9}$$

$$\left(\frac{\mu}{k}+\frac{\mu^*}{r^2}\right)u^*=\mu^*\left(\frac{1}{r}\frac{\partial u^*}{\partial r}+\frac{\partial^2 u^*}{\partial r^2}+\frac{\partial^2 u^*}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2 u^*}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v^*}{\partial \theta}\right)-\frac{1}{r}\frac{\partial p^*}{\partial \theta} \tag{10}$$

$$w^*=k\frac{\mu^*}{\mu}\left(\frac{1}{r}\frac{\partial w^*}{\partial r}+\frac{\partial^2 w^*}{\partial r^2}+\frac{\partial^2 w^*}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2 w^*}{\partial \theta^2}\right)-\frac{k}{\mu}\frac{\partial p^*}{\partial z} \tag{11}$$

Due to continuity of fluid in the porous elastic bearing, p^* satisfies the Laplace equation:

$$\frac{\partial^2 p^*}{\partial r^2}+\frac{1}{r}\frac{\partial p^*}{\partial r}+\frac{1}{r}\frac{\partial^2 p^*}{\partial \theta^2}+\frac{\partial^2 p^*}{\partial z^2}=0 \tag{12}$$

where p^*, k, μ^*, v^*, u^* and w^* are respectively pressure, permeability, effective viscosity, velocity components in r, θ and z directions, respectively, in the porous elastic bearing.

3 Boundary conditions

The boundary conditions associated to the system of equations (5), (9), (10), (11) and (12) are:

- On the symmetrical axis, $z=0$:

$$\frac{\partial p}{\partial z}=\frac{\partial p^*}{\partial z}=0\,,\ \frac{\partial u^*}{\partial z}=\frac{\partial v^*}{\partial z}=w^*=0 \tag{13}$$

- At edge of the bearing, $z=L/2$:

$$p=p^*=0\,,\ \frac{\partial u^*}{\partial z}=\frac{\partial v^*}{\partial z}=\frac{\partial w^*}{\partial z}=0 \tag{14}$$

- At the external bearing surface, $r=R+C+H_0$:

$$\frac{\partial p^*}{\partial r}=0\,,\ u^*=v^*=w^*=0 \tag{15}$$

- On the film-porous elastic bearing interface, $r=R+C+H_0-H$:

$$p=p^*,\ u=u^*,\ v=-v^*,\ w=w^* \tag{16}$$

$$\mu\frac{\partial u}{\partial y}-\eta\frac{\partial^3 u}{\partial y^3}=-\mu^*\left(r\frac{\partial}{\partial r}\left(\frac{u^*}{r}\right)+\frac{1}{r}\frac{\partial v^*}{\partial \theta}\right),\ \frac{\partial v^*}{\partial r}=0\,,\ \mu\frac{\partial w}{\partial y}-\eta\frac{\partial^3 w}{\partial y^3}=-\mu^*\left(\frac{\partial w^*}{\partial r}+\frac{\partial v^*}{\partial z}\right) \tag{17}$$

In the circumferential direction θ, the Reynolds boundary conditions [8] are adopted in the film fluid, and the boundary conditions in the porous elastic bearing are:

$$p^*(r,\theta=0,z)=p^*(r,\theta=2\pi R,z)=0 \tag{18}$$

4 Numerical resolution

The modified Reynolds equation in the film (5), those of Darcy – Brinkman equations (9) – (11) and Laplace equation (12) in the porous elastic bearing are interlinked at the film-bearing interface by the continuity of pressure, velocities and viscous shear stresses conditions. This fluid – porous bearing interaction problem cannot be solved analytically. To obtain approximate solutions, we use the finite differences method which approximates the differential equations by a system of algebraic equations. In order to easily perform a finite differences discretization, the physical domain is thus transformed into a rectangular one and non-dimensional variables are introduced.

5 Results and discussion

Figure 2 displays the comparison of dimensionless load capacity $\bar{w}$ predicted using BM and SFM with regards to two couple stresses parameters $\bar{l}$ at $\varphi = 2.10^{-4}$ and $C_0 = 0.01$. It is found that the load capacity prediced by BM is heigher than that of SFM. Physically, the Brinkman equation involves a viscous shear term, matching the shearing stresses by continuity condition on the porous elastic bearing interface, it resitsts the fluid motion and thus gives a significant contribution to $\bar{w}$. This difference becomes more pronounced when $\bar{l}$ or ε increses. This finding agrees with the results reported by Bujurke and Naduvinamani [2] for short porous journal bearings. In addition, increasing $\bar{l}$ yield to greater increment on $\bar{w}$ especially at high ε for both models. Couple stresses because of the lubricant mixed with additives oppose fluid motion and after that improve $\bar{w}$.

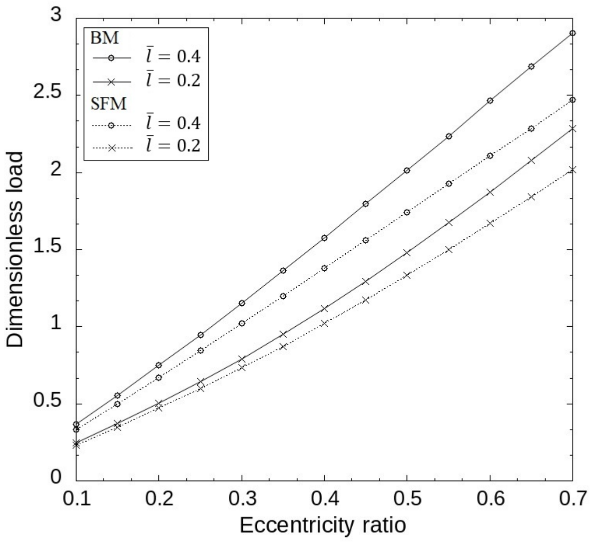

Figure 2 : Couple stress effect on dimensionless load capacity $\bar{w}$ at $\varphi = 2.10^{-4}$ and $C_0 = 0.01$

The comparaison of dimensionless load capacity $\bar{w}$ simulated by BM and SFM for two bearing permeability parameters φ at $\bar{l} = 0.2$ and $C_0 = 0.01$ is portrayed on Figure 3. It is found that the viscous shear effects of the BM provides an augmentation in $\bar{w}$ as compared to that obtained from the SFM. This increasing effect is more significant when the bearing operates under higher ε. It is seen that the impact of φ is to diminish $\bar{w}$ for both models. The decrement of this later is more enlarger for high ε. This is clarified by that the penetration ingoing the porous interface becomes easier with increasing φ. The load capacity $\bar{w}$ simulated by BM and SFM using two elastic coefficients C_0 at $\bar{l} = 0.2$ and $\varphi = 2.10^{-4}$ is depicted on Figure 4. The curves of these two models are observed to evolve with similar way. The BM provides higher $\bar{w}$ as compared to SFM, this influence is clearer when the eccentricity of the bearing is higher. The impact of variation of C_0 on $\bar{w}$ is sharply felt when the bearing operates under higher eccentricities ($\varepsilon > 0.65$). The elastic deformation of the film-porous bearing interface tends to increase the fluid film thicknesses, this explains the decrease in $\bar{w}$. On the whole the SFM underestimates the load capacity as compared to that obtained from BM. This trend was also found by Lin and Hwang [9] in the case of Newtonian fluid. Introducing the viscous shear term, the BM model considers more correctly the resistance encountered by the fluid flowing toward the porous elastic bearing and thus predicts a more realistic load $\bar{w}$.

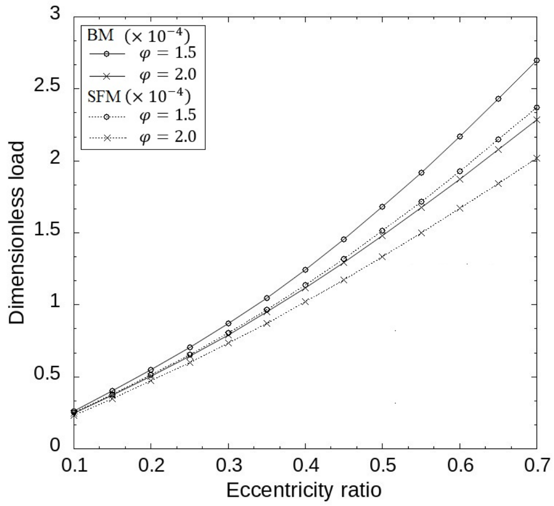

Figure 3 : Permeability parameters effect on dimensionless load capacity $\bar{w}$ at $\bar{l} = 0.2$ and $C_0 = 0.01$

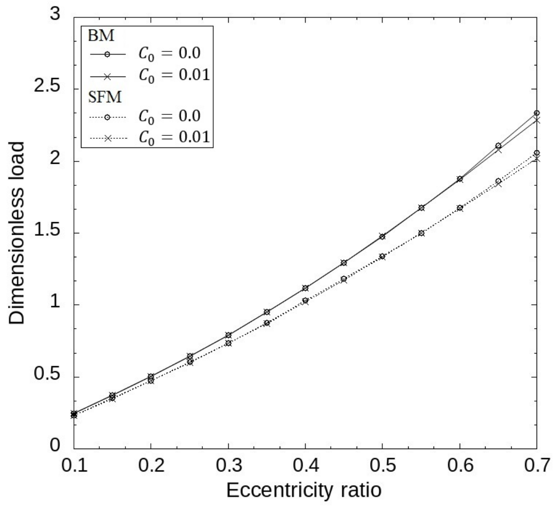

Figure 4 : Elastic coefficients effect on dimensionless load capacity $\bar{w}$ at $\bar{l} = 0.2$ and $\varphi = 2.10^{-4}$

6 Conclusion

In this paper, viscous shearing forces effects on finite porous elastic journal bearings were numerically analyzed. Making use of the Stokes micro-continuum mechanics, and considering bearing elastic deformation determined by Winkler model, an EHD modified Reynolds equation type is derived. The porous flow was modeled by the complete Darcy-Brinkman equation. According to the results, the viscous shearing forces effects of the Brinkman model BM on the load capacity are not negligible. Comparing with those obtained from Darcy model with Beavers-Joseph slip conditions SFM, the viscous shearing forces effects enhance this quantity. It is found that for both models, the couple stresses increase the load capacity, especially when the bearing operates under higher eccentricities. Furthermore, the bearing permeability parameter reduces it. The deformation impact on load capacity is sharply felt when the bearing operates under higher eccentricities.

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