New numerical tools for parameter identification from full-field measurements

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Abstract :

In this paper, we introduce a new robust procedure to identify material parameters of mechanical models from full-field measurements. It is based on data information coming from the Digital Image Correlation technique. The procedure aims at defining a suitable numerical processing, in terms of model selection and discretization mesh, with respect to information and noise contained in the data. The nature of the procedure is to minimize a cost functional based on the modified Constitutive Relation Error concept, which is made of modeling and measurement terms. Constructing an admissible stress field, verifying the equilibration equation in a full sense, enables one to obtain estimates on both discretization and modeling errors, which can then be compared with measurement noise in order to drive mesh adaptation and model enrichment. In addition, the procedure is coupled with reduced order modeling techniques in order to optimize computation costs. The overall approach is implemented on several numerical experiments with linear or nonlinear material behaviors.

Key words : parameter identification; digital image correlation; measurement noise; modified constitutive relation error; full sense equilibration; reduced order modeling.

1 Introduction

Parameter identification methods have become increasingly sophisticated and powerful in solid mechanics in order to qualify material behaviors at best and feed numerical simulations. Among the large scope of inverse identification methods, we focus here on those based on full-field measurements, i.e. performed from Digital Image Correlation (DIC) using a single camera [14], stereo image correlation (stereo-DIC) using several cameras [12], or digital volume correlation using X-ray microtomography or magnetic resonance imaging [4]. However, due to the large and noisy amount of experimental data that may be obtained from full-field measurements, an important and actual challenge is to extract from these data the part of information which is relevant for the identification objective and to take the measurement perturbations into account. In addition, it is important to use a method which remains robust with respect to perturbations.

Among identification methods adapted or fully dedicated to the use of full-field measurements, the most popular ones are Finite Element Model Updating (FEMU) [19], Equilibrium Gap Method [9], Reciprocity Gap Method [5], Virtual Fields Method [13], and Constitutive Relation Error (CRE) [11]. Despite their advances, several questions that have influence on the identification accuracy still attract attention. Particularly, two of important shortcomings of the above identification methods can be noted. Firstly, the measurement zone, over which the identification is performed, is usualy only a part of specimen surface. Therefore, the information about the remaining zone, especially on the border of specimen, may be lost. The lost information can contain reliable one, for instance, information on free edge or the measurement of global load applied to a larger zone than the measurement zone. Secondly, boundary conditions are needed for some standard indentification methods such as FEMU. When applying this type of method, if the calculation is performed only in the measurement zone, Dirichlet boundary condition must be defined by extracting from the measurements. Obtained boundary condition is always perturbed and this leads to measurement error. Otherwise, for a calculation performed in a larger zone, additional hypotheses on definition of boundary conditions are required, making appear additional modeling error. Therefore, it is essential to propose a method dedicated to full-field measurements which allows a calculation not limited to the measurement zone without additional hypotheses on the unavailable information and to take into account the measurement perturbations.

The modified CRE (mCRE) was applied for full-field measurements, particularly for identification of linear elastic material properties when dealing with uncertain data (the reliability of information on boundary conditions), in [2]. However, in that works, due to using of displacement formulation for the global identification problem, static admissibility of the stress fields was verified only in a weak way (finite element (FE) sense). That means the associated discretization errors were negligible compared to the modeling ones. Furthermore, covariance matrix of the measured displacement noise was not taken into account.

In [11], Florentin et al. proposed a new enhanced method based on CRE concept to solve the identification problem. The improvement of proposed method allows to facilitate the construction of admissible stress field without using FE stress field. The key points in this technique are choosing the finite dimension space of admissible stress field and technique to build that space. In this technique, admissible stress field is constructed based on the equilibrium at element level under the tractions on element edges that were assumed to be linear. The use of proposed method significantly improves the quality of the results and the method remains robust when multiple material properties are identified at the same time. However, the reliability of information on measurements and boundary conditions were not considered in that work.

In order to optimize computation costs, the procedure of constructing admissible stress field from FE solution can be coupled with one of the reduced order modeling techniques, Proper Generalized Decomposition (PGD), as shown in [1]. In this approach, geometry of all elements is paremeterized using a set of geomatrical parameters, then displacement field at element level is approximated by low-rank variable-separated one and the latter is obtained by means of progressive Galerkin PGD technique. This approach allows to find, for any configuration of the geometry and loading, the parameterized PGD displacement in an offline phase, then this solution is directy used in the online phase for each element of the mesh.

In present work, we propose a new method which is a mix of the method proposed in [2] and one proposed in [11]. Our method is based on the use of mCRE concept and the construction of the stress field which is admissible in full sense. Because the quality of the numerical model (discretization) becomes fundamental as now measurements are more accurate in space, we investigate the interplay between the mesh density and measurement accuracy. That means in our method discretization error will be taken into account and assessed. Comparing the latter and the modeling error will allow to have the optimal way to identify the model parameters by changing the model if the modeling error is much large than discretization one and/or adapting the mesh in the case of large discretization error. The final goal is to define a suitable numerical processing, in terms of model selection and discretization mesh, with respect to information and noise contained in the data.

The proposed method will be decribed in two parts. In the first part, mCRE will be used as the functional to be minimized to identify the model parameters. In this step, a displacement formulation of the problem will be used and discretization error will be ignored, similarly to the method in the works [2]. However, in our method, covariance matrix of measurement noise will be taken into accout. The method is described in the section 2 and numerical results are give in section 3.1. In the second part, an admissible stress field, verifying the equilibration equation in a full sense, will be constructed using PGD technique as described in [1]. Also, the latter will be used to get the optimal weighting factor that controls the balance between the two terms of mCRE as in [7]. All of points in the second part is discussed in section 3.2.

2 Methods

2.1 Reference problem

In this section, the 2D reference problem addressed in this work is defined. Consider a 2D specimen modeled as a 2D domain Ω with Lipschitz boundary $\partial\Omega$ that is split into 2 complementary parts: Dirichlet part, Γ_D, and Neumann one, Γ_N, such that: $\Gamma_D \cup \Gamma_N = \partial\Omega$, $\Gamma_D \cap \Gamma_N = \emptyset$, and $\Gamma_D \neq \emptyset$. The displacement measurement is perfomed all over the domain Ω and the continuous measured displacement field is denoted by $\underline{\tilde{u}}$. Assume the absence of body force, the systems of governing equations under the plane stress assumption writes:

$$\begin{cases} \text{Equilibrium:} & \underline{div}\ \underline{\underline{\sigma}} = \underline{0} & \text{in } \Omega & \text{(1a)} \\ \text{Kinematic compatibility:} & \underline{\underline{\epsilon}} = \frac{1}{2}(\nabla \underline{u} + \nabla^T \underline{u}) & \text{in } \Omega & \text{(1b)} \\ \text{Constitutive equation:} & \underline{\underline{\sigma}} = \mathbb{C}(\underline{p}) : \underline{\underline{\epsilon}} & \text{in } \Omega & \text{(1c)} \\ \text{Measurement:} & \underline{u} = \underline{\tilde{u}} & \text{in } \Omega & \text{(1d)} \\ \text{Dirichlet boundary condition:} & \underline{u} = \underline{0} & \text{in } \Gamma_D & \text{(1e)} \\ \text{Neumann boundary condition:} & \underline{\underline{\sigma}}.\underline{n} = \underline{f} & \text{in } \Gamma_N & \text{(1f)} \end{cases}$$

where $\underline{p}$ is a vector of model parameters to be identified.

The discrete measured displacement $\tilde{U}$ is often obtained by means of 2D-DIC techniques. For numerical applications, it is usually replaced by a synthetic one which is the sum of a free-of-error term U_0 and an added synthetic measurement error $\delta\tilde{U}$.

$$\tilde{U} = U_0 + \delta\tilde{U} \tag{2}$$

In this work, a reference FE displacement will be chosen for U_0 and a zero-mean Gaussian white noise whose standard deviation is set based on the mean value of U_0 will be chosen to represent the synthetic measurement error.

2.2 Identification problem formulation using the mCRE concept

MCRE is a variational approach. The philosophy is to enforce what is known on the model (equilibrium, sensor position...) and to relax what is not known (behavior, measurement values, boundary conditions potentially...); it thus refers to the concept of the reliability of information. It was initially introduced for dynamics models [8] then successfully applied in several applications with uncertain measurements or behaviors [10]. An extension to the nonlinear context was recently proposed in [17].

The mCRE concept leans on the CRE concept used for more than 40 years for the control of FEM calculations [16]. It is made of two parts: one measuring modeling error (CRE part), and one measuring the gap with measurements (of displacement, force...). The regularization is then performed from the model.

2.2.1 Continuous version

For linear elasticity, the CRE part of mCRE is an energy measure of the distance between an admissible stress field $\underline{\underline{\sigma}}$ and another stress field obtained from an admissible displacement field $\underline{u}$:

$$E^2_{CRE}(\underline{p}, \underline{u}, \underline{\underline{\sigma}}) = \frac{1}{2}\int_\Omega (\underline{\underline{\sigma}} - \mathbb{C} : \underline{\underline{\epsilon}}(\underline{u})) : \mathbb{C}^{-1} : (\underline{\underline{\sigma}} - \mathbb{C} : \underline{\underline{\epsilon}}(\underline{u}))d\Omega \tag{3}$$

For the first part of the proposed algorithm, a displacement formulation in which the admissible stress $\underline{\underline{\sigma}}$ is expessed as a function of a statically adimissible displacement field, $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}(\underline{v}) = \mathbb{C} : \underline{\underline{\epsilon}}(\underline{v})), \underline{v} \in H^1$, is used so that the CRE is the function of only model parameters and displacement fields $\underline{u}$ and $\underline{v}$:

$$E^2_{CRE}(\underline{p}, \underline{u}, \underline{v}) = \frac{1}{2}\int_\Omega \underline{\underline{\sigma}}(\underline{v} - \underline{u}) : \underline{\underline{\epsilon}}(\underline{v} - \underline{u})d\Omega \tag{4}$$

In this work, it is assumed that the information about force measurement in Neumann boundary is reliable. That means, the measurement part of mCRE consists of only one term that is the gap with the measured displacement $\underline{\tilde{u}}$. Therefore:

$$E^2_{mCRE}(\underline{p}, \underline{u}, \underline{v}) = E^2_{CRE}(\underline{p}, \underline{u}, \underline{v}) + \gamma\frac{1}{2}\eta||\underline{u} - \underline{\tilde{u}}||^2 \tag{5}$$

where η is a scaling coefficient which is chosen so that both terms are of the same unit. The weighting coefficient γ is often written as $\frac{r}{1-r}$. It may be set using the Morozov principle [18] or the L-curve method [6], and this choice should be made in regards of the à priori reliability on both model and measurements.

The identification problem is then defined as a minimization problem:

$$\underline{p}_{opt} = \arg\min_{\underline{p},\underline{u},\underline{v}} E^2_{mCRE}(\underline{p}, \underline{u}, \underline{v}) \tag{6}$$

In practice, this problem is solved in two steps. The first one, called localization step, is to build the addmissible stress and displacement fields for a given parameter $\underline{p}$. The second step, called correction step, consists in finding the best model parameters $\underline{p}_{opt}$ by minimizing a cost function derived from the solution of previous step.

$$\min_{\underline{p},\underline{u},\underline{v}} E^2_{mCRE}(\underline{p},\underline{u},\underline{v}) = \min_{\underline{p}} \min_{\underline{u},\underline{v}} E^2_{mCRE}(\underline{p},\underline{u},\underline{v}) \tag{7}$$

This leads to an iterative method, each iteration consisting of two partial minimization steps.

2.2.2 Discretized version

This continuous version will be discretized so that the problem can be solved numerically. By projecting the unknown displacement fields on classical FE shape functions over Ω, the discretized mCRE, denoted E^h_{mCRE}, can be obtained and reads:

$$E^{h2}_{mCRE}(\underline{p},U,V) = \frac{1}{2}(U-V)^T K(U-V) + \gamma\frac{1}{2}(U-U_{obs})^T G_u(U-U_{obs}) \tag{8}$$

where K is the global stiffness matrix. V is statically admissible in the sense that it verifies equilibrium equations in a FE sense (and therefore the associated discretization error is ignored): $K_{go}V = F_{g1}$ where "o" denotes the index of all DOFs and "g" denotes the index of all active DOFs (that means K_{go} is the submatrix obtained by removing all the rows corresponding to the prescribed DOFs). G_u is a scaling diagonal matrix that integrates η. When measurement noise is taken into account, the optimal value of weighting factor γ can be chosen by means of Morozov discrepancy principle or L-curve method. In this case, it is convinient to express matrix G_u in the following form:

$$G_u = \frac{1}{N_u^2}(U_0^T K_0 U_0) C_u^{-1} \tag{9}$$

In this expression, K_0 is the global stiffness matrix associated to reference model paremeters, C_u is the covariance matrix of displacement noise, and N_u is the number of DOFs of displacement. In the case of using DIC to measure displacement, due to the noise when acquiring reference and deformed images, C_u is normally not diagonal because the displacement noise is spatially correlated. In this work, displacement noise is spatially uncorrelated, therefore C_u is a diagonal matrix whose all the diagonal components are the same and equal to the variance of displacement noise. Furthermore, the prefactor $1/N_u^2$ is chosen so that, at convergence, the measurement term, normalized by covariance matrix of displacement noise, is of order 1. Thus, the discretized version of the identification problem writes:

$$\underline{p}_{opt} = \arg\min_{\underline{p},U,V} E^{h2}_{mCRE}(\underline{p},U,V) \tag{10}$$

2.3 Solution algorithm

2.3.1 Localization step

In this step, the functional $E^{h2}_{mCRE}(\underline{p}, U, V)$ will be minimized under constraint: $K_{go}V = F_{g1}$. It is a constrained minimization problem and we introduce the Lagrangian:

$$L(U, V, \Lambda) = \frac{1}{2}(U - V)^T K(U - V) + \gamma\frac{1}{2}(U - U_{obs})^T G_u(U - U_{obs}) - \Lambda^T(K_{go}V - F_{g1}) \quad (11)$$

where the Lagrange multiplier Λ is a column vector of g components. Finding its stationary point leads to the system:

$$\begin{cases} K(U - V) + \gamma G_u(U - \tilde{U}) = 0 & (12a)\\ K(U - V) + K^T_{go}\Lambda = 0 & (12b)\\ K_{go}V = F_{g1} & (12c)\end{cases}$$

By introducing a colum vector Λ^* of N_u components such that:

$$\begin{cases} \Lambda^*_{g1} = \Lambda & (13a)\\ \Lambda^*_{n1} = \emptyset_{n1} & (13b)\end{cases}$$

where n denotes the prescribed DOFs, we can note that:

$$\begin{cases} K^T_{go}\Lambda = K\Lambda^* & (14a)\\ K_{go}\Lambda^* = K_{gg}\Lambda & (14b)\end{cases}$$

Thus, the equation (12b) becomes:

$$K(U - V + \Lambda^*) = 0 \quad (15)$$

Equation (15) corresponds to a zero-Neumann boundary problem whose any solution is a rigid body motion. Among the solutions, a zero rigid body motion is chosen:

$$U - V + \Lambda^* = 0 \quad (16)$$

Taking equation (13b) into account, it can be deduced that the displacement fields U and V are the same at prescribed DOFs:

$$U_{n1} = V_{n1} \quad (17)$$

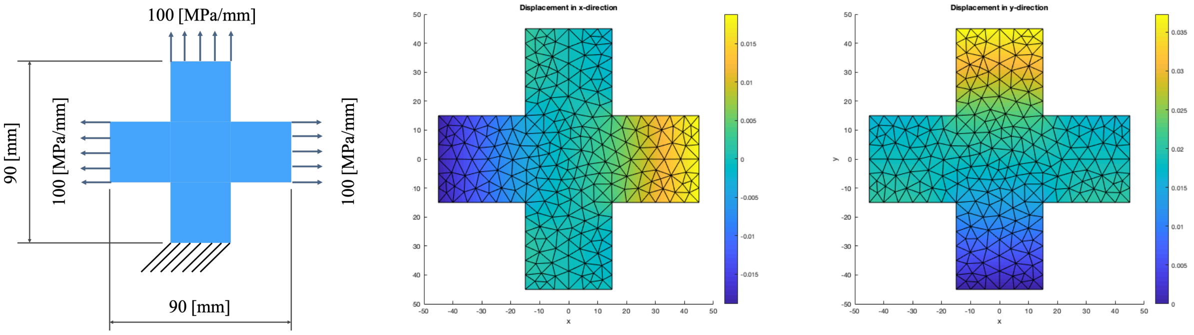
and:

$$U_{g1} - V_{g1} = \Lambda \quad (18)$$

By substituting all properties (14, 16, 18) into system (12), the solution (U, Λ) can be obtained from final matrix equation:

$$\begin{bmatrix} \gamma G_u & -K_{og}\\ K_{go} & K_{gg}\end{bmatrix}\begin{bmatrix} U\\ \Lambda\end{bmatrix} = \begin{bmatrix}\gamma G_u\tilde{U}\\ F_{g1}\end{bmatrix} \quad (19)$$

Finally, the displacement field V can be computed from (16).



(a) Problem description (b) Simulated displacement

Fig. 1. Numerical example of a plate

2.3.2 Correction step

The correction step is to solve an unconstrained optimization problem:

$$\min_{\underline{p}} F(\underline{p}) \text{ with } F(\underline{p}) = E^{h2}_{mCRE}(\underline{p}, U, V) \tag{20}$$

This is an nonlinear process that uses an optimization algorithm, such as the gradient method with optimal path: $\underline{p}^{(k+1)} = \underline{p}^{(k)} - \alpha^{(k)} gradF(\underline{p}^{(k)})$. The gradient of $F(\underline{p})$ can be explicitly computed with low computational effort as shown in [2]. Particularly, the derivative of $F(\underline{p})$ with respect to any model parameter p_i is:

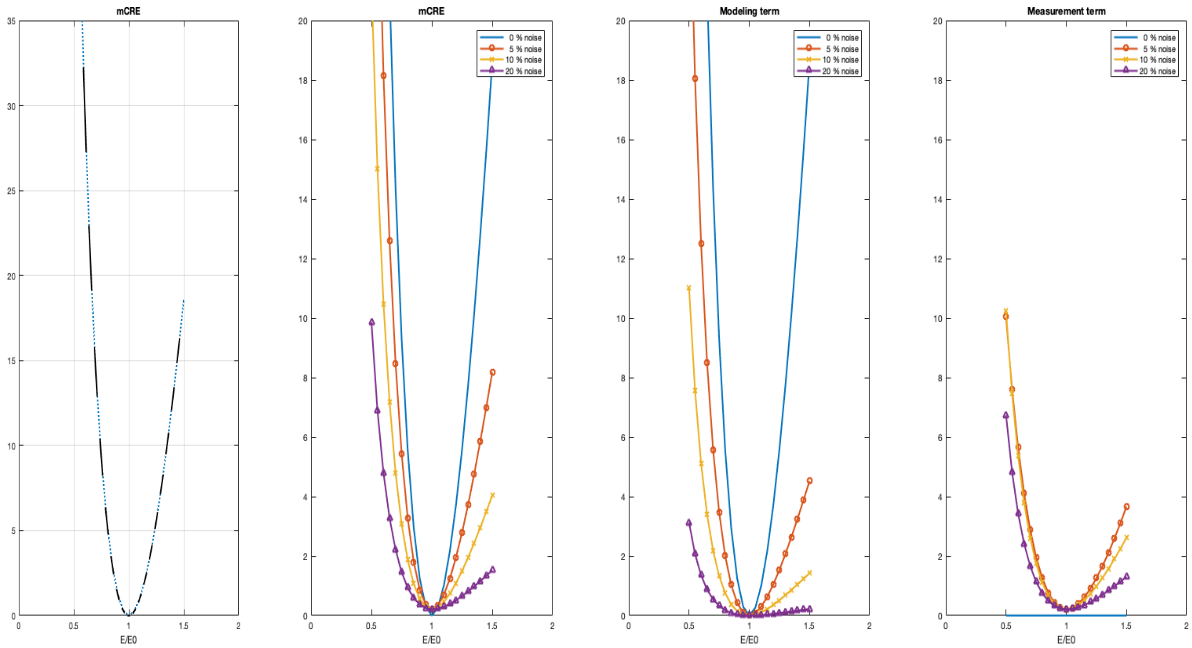
$$\frac{\partial F(\underline{p})}{\partial p_i} = \frac{1}{2}(U-V)^T \frac{\partial K}{\partial p_i}(U-V) - \Lambda^T \frac{\partial K_{go}}{\partial p_i} V = \frac{1}{2}(U-V)^T \frac{\partial K}{\partial p_i}(U+V) \tag{21}$$

As expected, the gradient vanishes when $U = V$. This confirms that exact solution is minimizer of the mCRE and this solution verifies all equations of the problem. In practice, the iterative process is stopped when the cost function $F(\underline{p})$ reaches a given tolerance value.

3 Results and discussion

3.1 Results

In this section, the illustration of the method on a numerical example based on the identification of the Young modulus of a cross-shaped plate is proposed. The plate is assumed to be isotropic whose behavior is described by Young modulus and Poisson ratio. The plate is clamped on its bottom side and is subjected to the uniform distributed loads at top, left, and right sides (Fig. 1a). At first, the simulated displacement field U_0 is computed for a given reference set of Young modulus and Poisson ratio ($E_0 = 200000$ MPa and $\nu_0 = 0.26$) on a FE mesh, generated by GMSH, containing 261 nodes and 448 elements (Fig. 1b). Then, the measured displacement field $\tilde{U}$ is generated on a FE mesh (meaning that the data grid is the same as FE mesh) by adding to the simulated displacement a zero-mean Gaussian white noise $\delta\tilde{U}$ whose standard deviation, called noise level, is chosen based on mean value of simulated displacement. Different noise levels are considered in order to evaluate the robustness of the method. For this part of algorithm, the initial choice of weighting factor $\gamma = 1$ is performed which leads to relevant results as shown in [2].



(a) mCRE and its gradient without noise

(b) mCRE (left) and its modeling (center) and measurement (left) terms for different noise levels

Fig. 2. Numerical example of a plate

Fig. 2a shows the shape of mCRE functional and its gradient with respect to Young modulus at different points as a function of the ratio E/E_0 without noise. The segments perfectly match the slope of mCRE functional which confirms the good estimation of the gradient. Fig. 2b shows the shape of mCRE functional and its two terms for different noise levels. It can be seen that the algorithm is consistent and robust with respect to the noise: the optimal values of Young modulus, which is minimizer of mCRE functionals, are very close to the reference one; and the measurement error vanishes if there is no measurement noise. Furthermore, the two terms of mCRE functional are of the same order and, thanks to normalization of the measurement term by noise covariance matrix, are close to 1.

3.2 Discussion

The mCRE is very flexible, we can relax everything which is not known in the identification procedure. In the measurement term, we include a weight with covariance matrix to take measurement noise into account. To fully describe the proposed method, some important points will be discussed in following sections.

3.2.1 Choosing the optimal weighting factor

The role of weighting factor γ is to control the balance between the two terms of the mCRE functional. Depending on the value of the weighting factor, model or measurements are more or less taken into account in the formulation. As minimizing mCRE is a nonlinear process that is solved by an iterative method (gradient descent one in this work), this factor has important fluence on the values of both terms and the balance between them. Therefore, an optimal choise of γ is of great importance. In recent works, this can be done by using Morozov's discrepancy principle [18] or L-curve method [6]. In our method, in order to optimize computation costs, PGD technique will be used. This point will be discussed in section 3.2.4.

3.2.2 Assessment of modeling and discretization errors

Up to now, the identification problem was solved without taking the discretization error into account. This can be done only in the case the latter is negligible with respect to the modeling error. In the second part of our method, discretization error will be taken into account and it will be an error estimator based on that the mesh will be refined. By this end, full error, which is the sum of modeling error and discretization one, will be computed as following:

$$E_{CRE}^{f2}(\underline{p}, \underline{u}, \underline{\underline{\hat{\sigma}}}) = \frac{1}{2}\int_{\Omega}(\underline{\underline{\hat{\sigma}}} - \mathbb{C} : \underline{\underline{\epsilon}}(\underline{u})) : \mathbb{C}^{-1} : (\underline{\underline{\hat{\sigma}}} - \mathbb{C} : \underline{\underline{\epsilon}}(\underline{u}))d\Omega \qquad (22)$$

where $\hat{\sigma}$ is the fully admissible stress field. This stress field can be constructed from FE stress by means of classical techniques, for examples hybrid-flux or EET, or constructed without using FE stress by using techniques developed in [1].

Once full error and modeling one are known, discretization error can be obtained as following:

$$E_{CRE}^{d2}(\underline{p}, \underline{u}, \underline{\underline{\hat{\sigma}}}) = E_{CRE}^{f2}(\underline{p}, \underline{u}, \underline{\underline{\hat{\sigma}}}) - \frac{1}{2}(U - V)^T K (U - V) \qquad (23)$$

3.2.3 Mesh adaptation and/or Model change

Mesh adaptation for inverse problems was investigated in [3]. In the context of full-field measurements, the CRE was used to assess the accuracy of interpolation of boundary conditions in [15]. In [20], an approach is developed to fulfill the desire of minimizing the error due to discretization, caused by meshing and refinement based on the users instinct. By using an adaptive mesh, locations where the displacement is rather heterogeneous are automatically refined by an algorithm similar to FE analysis. This is the first development of a fully self-adaptive global image correlation algorithm. To decide to perform mesh adaptation and/or model change, discretization and modeling errors are compared. If the modeling error is much large than discretization one, the model will be replaced by a more complex one (for example, model with isotropic material will be replaced by new model with orthotropic or even anisotropic material). Inversely, in the case of large discretization error, adapting the mesh will be performed. If the two errors are of the same order but their values are too large, mesh adaptation and model change can be performed at the same time.

3.2.4 Using of PGD technique

As shown in [1], when using PGD technique to find parameterized PGD displacement at element level, set of geometrical parameters of all element of the mesh plays the role of additional variable. However, thanks to separation of the solution into product of one-variable functions, the process of solving the element equilibration problem is still much faster. Therefore, addition variable does not have important influence on the computation cost.

PGD technique can also be used to optimize the value of weighting factor γ, as shown in [7]. By this end, weighting factor is considered as an addition variable of mCRE function, and the latter is optimized with respect to not only U, V, and $\underline{p}$, but also γ. Doing this, the minimization problem will be solved once for all potential values of γ and its optimal value can be found quickly with cheaper computation cost.

4 Conclusions

We proposed a method to identify material parameters of mechanical models from full-field measurements, taking measurement noise into account, based on mCRE concept and using PGD technique. In the proposed method, model parameters to be identified is the minimizer of mCRE that consists of modeling error (CRE term) and weighted measurement one. In the first part of the proposed method, the discretization error was ignored and the weighting factor was chosen à priori. The minimization problem was solved by iterative method in which each iteration is split into two steps: the first localization step is a constrained minimization problem that is solved by means of Lagrange multiplier; the second correction step is an unconstrained minimization problem solved by using gradient descend method. Numerically simulated results showed the consistence and the robustness of the proposed method with respect to noise. In the second part of the proposed method, the discretization error will be taken into account and then PGD technique will be used to solve the problem (including optimizing weighting factor γ) in order to optimize the computation costs. Finally, discretization and modeling errors will be assessed to define best model and optimal mesh with respect to information and noise in the data. All these points were discussed and will be the topic of forthcoming research works.

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