Enhancing and controlling parametric instabilities in structures : Application to an electromagnetic pendulum.

Arnaud Lazarus[a], Alvaro Anzoleaga Grandi[a], Suzie Protière[a]

a. Institut Jean Le Rond d'Alembert (IJLRA), CNRS : UMR7190, Sorbonne Université UPMC Paris VI, France, arnaud.lazarus@sorbonne-universite.fr

Résumé :

Les instabilités paramétriques peuvent survenir lorsque l'état mécanique d'une structure est modulé périodiquement dans le temps. Ces instabilités sont souvent traitées comme des phénomènes à éviter par exemple en génie maritime (roulement paramétrique) mais elles ont parfois été exploitées en mécanique des fluides (ondes de Faraday) ou dans les systèmes microélectromécaniques (amplification paramétrique). Une limitation bien connue pour exploiter pleinement les instabilités paramétriques classiques basées sur une faible modulation périodique d'un état mécanique est que les forces de friction internes attenuent très rapidement les résonances paramétriques sous-harmoniques. Pour palier à ces limitations, nous proposons une approche originale qui consiste à moduler de façon extrême l'état mécanique d'un système, en passant d'un état de vibration libre à un état quasiment divergent. Ce nouveau concept vibratoire est illustré ici à travers l'analyse numérique et expérimentale d'un pendule électromagnétique. Nous trouvons qu'il est possible non seulement d'augmenter considérablement le nombre de régions d'instabilité sous-harmoniques mais aussi de controller la taille des zones de stabilité, ouvrant ainsi une voie prometteuse pour la récupération d'énergie large bande et la discrétisation spectrale dans les systèmes vibratoires.

Abstract :

Parametric instabilities are dynamical instabilities arising when the mechanical state of a structure is properly modulated in time. It is sometimes seen as a phenomenon to avoid for example with sailing ships (parametric rolling) but it has also been exploited in vibrating fluids (Faraday waves) or NanoElectro-Mechanical Systems (parametric amplification). One well-known limitation in fully exploiting classic parametric instabilities based on small periodic modulation of a mechanical state is that inherent friction forces rapidly cancel sub-harmonic parametric resonances. To overcome this drawback, we suggest to formerly modify the state of a mechanical system close to its diverging instability. This original way of enhancing and controlling parametric instabilities is illustrated here through the numerical and experimental implementation of an electromagnetic pendulum. Not only we find it is possible to greatly enhance the number of subharmonic instability regions, but it is feasible to control the width of those regions, opening a promising way for frequency filtering in NanoElectroMechanical Systems.

Keywords : Parametric instabilities, structural vibrations, experimental vibrations, numerical models.

1 Introduction

Nowadays, from the use of buckling for folding [1] to the exploitation of fluttering piezoelectric flags for energy harvesting [2] or the benefit of parametric resonances for the reduction of parasitic signals in microelectromechanical systems [3], elastic instabilities eventually occurring in slender elastic structures are often seen as an opportunity to seize rather than a failure to avoid. Parametric instabilities, found in many engineering problems [4, 5], can be caused by the self-synchronized periodic modulations of the elastic state of a slender structure [6]. Although promising for elaborate functionalities, the optimal use of parametric instabilities in elastic structures is often restricted to micro or nano- electromechanical systems (MEMS or NEMS) where damping is sufficiently low for the principal and subharmonic instability regions to exist [7]. To overcome this drawback and fully exploit the potential of parametric instabilities for functionality at any scales, a change of paradigm is necessary : instead of classically lowering damping to favor parametric excitations from the small modulations of an elastic state, one could periodically impose a drastic change of elastic state to enhance dynamical instabilities at common damping (see Fig. 1).

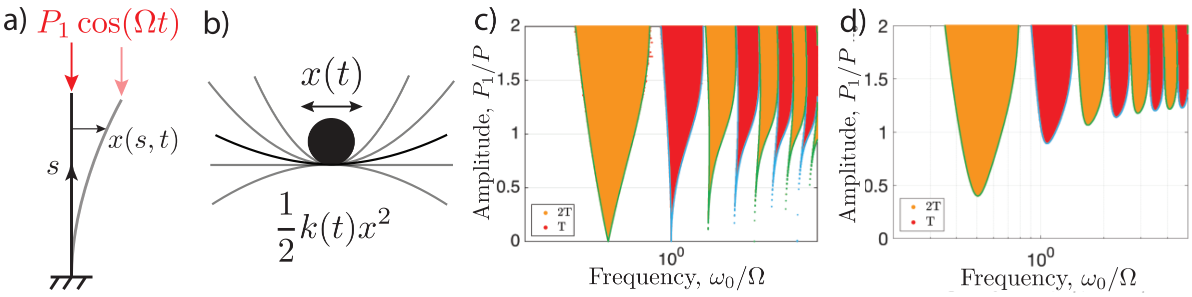

FIGURE 1 – Modulation of periodic elastic state. a) The Bolotin column : a cantilever beam under a conservative periodic compressive force is an archetypal example of a structure in periodic elastic state. b) When reduced on its first vibrational mode, the structure in a) can be modeled as a mass moving in a periodically time-varying quadratic energy potential. c-d) Stability charts depicting the famous Mathieu's tongues illustrating the zone of the modulation parameters space where parametric instabilities occurs. Yellow and red colors show parametric instability regions of period $2T = 4\pi/\Omega$ or $T = 2\pi/\Omega$, respectively. No colors indicate the straight beam of figure a) is neutrally stable. c) With almost no damping, parametric instabilities emerge even at low amplitude modulation, a property that has been exploited in MEMS [3]. d) With no damping however, parametric instabilities mostly disappear unless one finds a way to deal with very large modulation amplitude.

2 System under study and results

Here, we conduct and present the first numerical and experimental dynamical system that allows us to explore and rationalize the concept of extreme periodic modulation. Our goal is to relatively easily periodically modulate a mechanical system between two very different mechanical states in order to enhance parametric instabilities even in the presence of classic internal friction forces. To achieve our goal, we set up in the lab the electromagnetic pendulum shown in Fig.2a). The experiment consists of a magnetic pendulum that is symmetrically placed between two attracting electromagnets. When the electromagnets are off, the system is a simple pendulum characterized by a natural frequency $\omega_0 \approx 9$ rad/s as illustrated in the experimental plot of Fig.2c). When turning the electromagnets on by inputing an electrical current I, the mechanical state of the pendulum can be drastically modified. In our example of Fig.2, when the control parameter I is slightly below $I_{max} = 1.15$ A, our system is now naturally oscillating with a slower natural frequency. And above $I = I_{max}$ (our diverging threshold), our system is no more oscillating but diverging, on the right or left electromagnet depending on the imperfections in our system. This mechanical system is therefore a simple first realization of what we coined an extreme parametric oscillator, because with a single parameter I, we are drastically changing the state of our system, eventually from an oscillating to a diverging state. In classical parametric oscillators (parametric pendulums, Faraday's experiments, parametric rolling, etc...), this extreme modulation is hardly reachable because the geometrical of mechanical modulation parameters that come into play (length, gravity, height of the waves, etc...) are not easily varied on such scales.

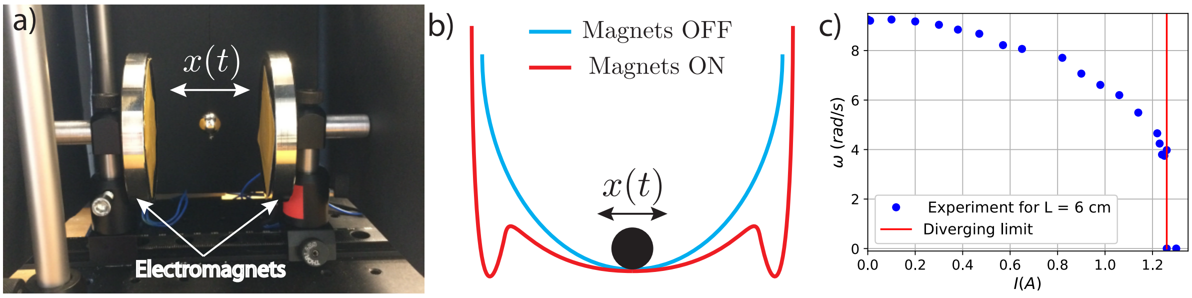

FIGURE 2 – The electromagnetic parametric oscillator under study. a) A pendulum whose mass is made of steel, is symmetrically placed between two identical attracting electromagnets that are periodically turned on (red energy states in b)) and off (blue energy states in b)). Because the symmetric energy landscape varies drastically, parametric instabilities are enhanced. b) Simplified "Particle in a time-varying potential well" model. c) Evolution of the natural frequency of the pendulum for various value of the electrical current I in the two electromagnets when the laters are separated by $L = 6$ cm. Below $I < I_{max} \approx 1.25$ A, the pendulum is naturally oscillating if perturbed. Eventually, for I close to the diverging limit I_{max}, the natural frequency goes down until it reaches zero, i.e. the mass is no more oscillating but diverging.

The dynamical behavior of our extreme parametric oscillator of Fig.2 is given in the preliminary experiments of Fig.3 that consist in periodically turning the electromagnets on or off in a square wave fashion as illustrated in Fig.3a). Fig.3b) shows the experimental stability charts of our electromagnetic pendulum in the modulation parameters space : period of modulation and amplitude of modulation as depicted in Fig.3a). The blue triangles indicate that the pendulum is dynamically stable, i.e. that the pendulum may be slightly vibrating but stay close to the trivial vertical state, in the middle of the electromagnetic cell. The crosses indicate the modulation parameters for which the mass was parametrically unstable, i.e. dynamically impacting the electromagnets. The color legend indicates the number of cycles the pendulum is doing in the emerging nonlinear vibrational regime. Modes with an integer number of M represent T-periodic unstable regions when the other M numbers represent $2T$ unstable regions. It is interesting to note the effect of the extreme parametric modulation. For relatively low modulation amplitude, $I/I_{max} < 0.8$, the pendulum is often stable, except eventually for the first ($M = 0.5$) or second ($M = 1$) instability regions (sometimes the third one $M = 1.5$ is observed). But for higher modulation amplitude close to the diverging threshold such as $I/I_{max} = 0.91$, it is possible to trigger highly sub-harmonic instability regions, here up to the 58^{th} instability regions ($M = 29$, not shown in the figure) when the current record demonstrated in a microelectromechanical device was found to be 28^{th} [3] ! Since it is possible to trigger vibrational motion with a large range and low values of parametric excitation frequencies, two-states oscillators, or extreme parametric excitation, could be promising for very large-band energy harvesting devices. Note finally that in parallel, a simple model of linear pa-

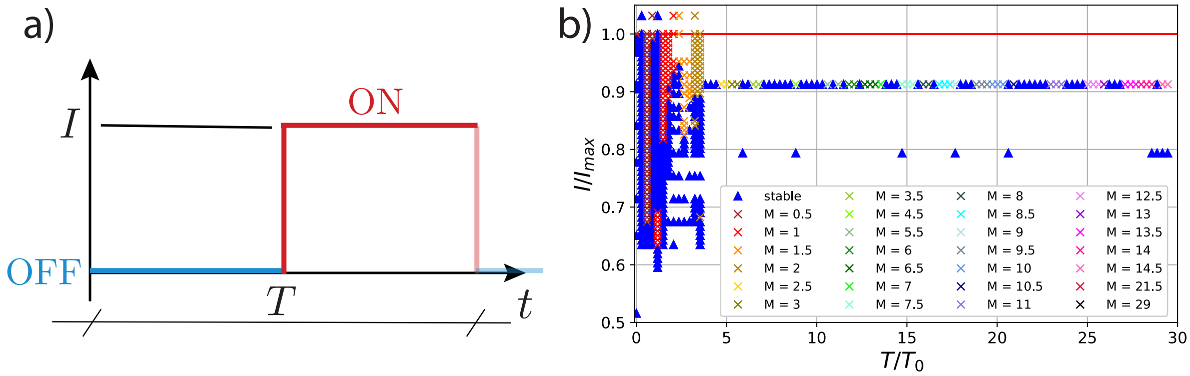

Figure 3 – Pendulum in a time-varying electromagnetic field. a) The electromagnets are now turned on and off in a square wave fashion with period T. The amplitude of modulation is the electrical current I when the electromagnets are on. b) Stability chart of the pendulum in the dimensionless modulation parameter space T/T_0 and I/I_{max} where T_0 and I_{max} represent the natural period when the electromagnets are off and the electrical current at static diverging threshold, respectively. Blue triangles represent stable states where the mass stays more or less in the middle of the electromagnets. Crosses represent unstable states where the pendulum dynamically diverges to eventually impact the electromagnets. The M numbers indicate that the mass is doing $M \times T_0$ cycles during the half period when the electromagnets are off.

rametric oscillator has been developed to gain physical insight in the experiments of Fig.3 and establish simple design laws to characterize the effect of extreme parametric modulation on parametric instabilities. Also, the choice of a square wave signal as modulation function is justified and exploited owing to the fact that a parametric oscillator with a step modulation function can be modeled by a Meissner equation [8] for which dynamical stability charts are completely known analytically.

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