Direct identification of Fung model parameters using evolutionary optimization approach

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Résumé :

Ce travail concerne l'identification numérique du modèle rhéologique complexe (anisotrope-hyperélastique) de Fung. Un tel modèle n'est pas explicitement convexe et nécessite donc la prise en compte de conditions supplémentaires, qui ne peuvent pas être introduites dans un processus d'identification classique. Aussi, une procédure d'optimisation spécifique, basée sur une approche évolutionnaire, a été développée. Cette stratégie d'identification a été validée via des résultats expérimentaux de la littérature concernant un tissu biologique, ainsi que pour un matériau tissé hautement orthotrope.

Abstract :

This work is dedicated to the numerical identification of the complex rheological anisotropic-hyperelastic Fung model. Such model is not explicitly convex, and so requires taking into account additional conditions, that cannot be introduced in a classical identification process. Thus, a specific optimization procedure, based on evolutionary approach, has been developed. This identification strategy has been validated by comparisons with experimental results of the literature concerning a biological tissue, and also for a highly orthotropic woven material.

Keywords: Identification; Hyperelastic-anisotropic model; Convexity conditions; Evolutionary optimization

1 Introduction

The numerical identification of hyperelastic constitutive models parameters is a common problem that reveals often in finite element (FE) simulations. For such complex models, a basic step-by-step classical identification through experimental stress-strain curves could not be performed [1]. Moreover, for numerous models as observed for Fung model [2], some non-explicit convexity conditions, greatly influenced by the chosen material parameters, should be taken into account during the identification process [3]; the omission of these conditions could lead to non-realistic results and instabilities in the numerical FE implementations. Thus, the integration of these convexity conditions

in a process of rheological parameter direct identification represents a great difficulty. It therefore requires specific identification tools based on a global optimization procedure.
In the present work, we focused on the identification of the parameters of an orthotropic woven tissue using Fung hyperelastic-anisotropic model. We proposed a direct numerical identification process, taking directly into account the convexity conditions, permitting to determine the corresponding material parameters. The developed identification tool is based on the evolutionary global optimization approach, using a Genetic Algorithm (GA). This identification strategy thus allows establishing parameter identifications performed simultaneously on several experimental stress-stretch curves, in the warp and the weft directions, which is substantial for the identification of anisotropic materials.
The effectiveness of our identification approach has been first tested for an anisotropic hyperelastic biological tissue from the literature. Next, the accuracy and reproducibility of this identification method have been evaluated by comparing the deduced analytical stress-stretch response with the experimental curves of the studied orthotropic woven tissue.

2 Fung model

Most of biological soft tissues such as soft cartilages, arteries and veins exhibit specific hyperelastic anisotropic behaviors. Many energy functions based on different approaches were proposed in order to capture the response of these biological materials [2, 4]: the polynomial [5], logarithm [6] and exponential [4] approaches. In this paper, we are especially interested to the exponential approach, proposed by Fung *et al* [2], implemented in ABAQUS FE software. Initially introduced for two-dimensional exponential form, the Fung model was developed in order to give an approximation for an artery subjected to internal pressure and longitudinal stretching while considering the anisotropic property of arteries and the nonlinearity of stress-strain curve. The model was later generalized to arbitrary three-dimensional states by Humphrey (1995) [7].
The strain energy function of Fung model is formulated as (with W : volumetric strain energy; $\mathbf{E}$: Green Lagrange strain tensor ; D : reverse bulk modulus; J : Jacobian) [2, 4]:

$$W = \frac{C}{2}\left(\exp(Q)-1\right)+\frac{1}{D}\left(\frac{J^2-1}{2}\right) \qquad (1)$$

where Q can be expressed in the principal basis using the following expression:

$$Q = B_{11}E_{11}^2 + B_{22}E_{22}^2 + B_{33}E_{33}^2 + 2B_{12}E_{11}E_{22} + 2B_{13}E_{11}E_{33} + 2B_{23}E_{22}E_{33} \qquad (2)$$

Thus, for incompressible materials, seven parameters control the model:

$$X = \{C, B_{11}, B_{22}, B_{33}, B_{12}, B_{13}, B_{23}\} \qquad (3)$$

As detailed in [3], this model is not explicitly convex. To ensure realistic analytical solutions as well as numerical stability for FE calculations, it is thus necessary to verify three additional conditions expressed by:

$$C > 0 \quad ; \quad B_{ii} > B_{ij} \quad ; \quad \left(B_{ii}, B_{ij}\right) > 0 \qquad i, j = 1:3 \qquad (4)$$

3 Identification strategy

3.1 Formulation of the identification problem

Considering a set of experimental stress-stretch curves (each comprising $N_{p,i}$ experimental points $P_{i,j}^{Exp}$), the identification of the seven Fung parameters (see Eq.(3)) could be expressed as an optimization problem [1]:

$$\text{Find } X \text{ minimizing the objective function } f(X) = \left(1 + \sum_{i=1}^{2}\left(\frac{1}{N_{p,i}}\sum_{j=1}^{N_{p,i}}\left|P_{i,j}^{Num} - P_{i,j}^{Exp}\right|\right)\right) \tag{5}$$

the $P_{i,j}^{Num}$ values being deduced from analytical calculations.

Moreover, the convexity conditions mentioned above lead to a particular difficulty: the convexity conditions expressed by Eq.(4) could not be taken into account explicitly in the expression of *f*. Therefore, before any objective function evaluation, a preliminary test of the convexity conditions should be performed to each *X* parameter set.

3.2 Optimization procedure

It should be reminded that such identification problem (see Eq.(5)) for complex rheological material models could not be performed using classical step-by-step approaches [1].

Some first trials have been carried out to identify numerically and directly the model parameters using standard local optimization tools implemented in MATLAB. However, for such procedures, the conditions of convexity (i.e. Eq. (4)) could not be integrated in the identification process: it has been observed that the successive runs lead to non-reproducible parameter sets *X*, and also unrealistic analytical solutions.

Indeed, the studied Fung parameter identification problem requires a global optimization procedure which also allows the pre-testing of convexity conditions. A Genetic Algorithm (GA), based on evolutionary-like process, was designed. Such procedure is a global, stochastic search method inspired from natural evolution [8]: it manipulates a fixed number (i.e. population) of potential solutions *X* (i.e. individuals) progressing along successive generations. The best solutions are intended to survive and to form the next generation by exchanging and mixing their characters, using different biological inspired operators: selection, crossover and mutation.

The developed GA manipulates directly real and integer variables. The corresponding genetic operators have been chosen following recommendations of the literature [1, 8]: the selection operates by tournaments (between randomly chosen individuals), and this selection is elitist (i.e. best individual automatically stored); the whole arithmetical crossover (i.e. barycentric arrangement between selected individuals) and the random uniform mutation are applied.

4 Application

The optimization tool was tested for two hyperelastic-anisotropic material identifications, using comparisons with experimental data. First, the parameters of Fung model have been searched to reproduce analytically the experimental results of Holpzafel *et al.* [9] concerning a biological tissue (material (A)): the experimental stress-stretch curves considered then correspond to the response of coronary artery adventitial layer, in the longitudinal (1) and circumferential (2) directions.

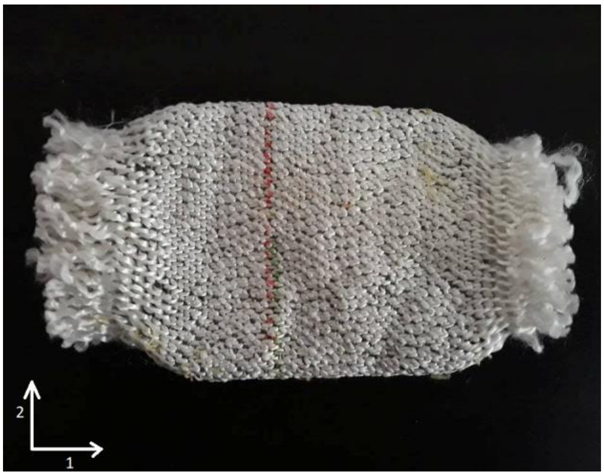


Figure 1: Structure of woven material

Fung model is generally dedicated to biological behaviors. However, in this work, this model was next tested to reproduce the response of a non-biological orthotropic tissue (material (B)): a woven material, shown in Fig.1, has been analyzed. This material is tubularly woven. The weft yarn is a gimped elasthane yarn, the covering being made with Vectran, which is a manufactured fiber, spun from a liquid-crystal polymer (LCP). The warp yarn is a standard polyester yarn. This composition gives the material extremely different deformability properties in the warp and weft directions, which results in a high degree of anisotropy. To identify the model parameters, uniaxial tensile tests in the warp (1) and weft (2) directions, have been preliminary performed at room temperature and under a crosshead speed of 10 mm/mn on rectangular strips of 85 mm length and 25 mm width. This geometry was chosen according to the standards existing for fabrics. The highly anisotropic behavior has been confirmed, the material exhibiting ultimate extension that reaches up to 40% in the longitudinal direction and up to 100% in transverse direction.

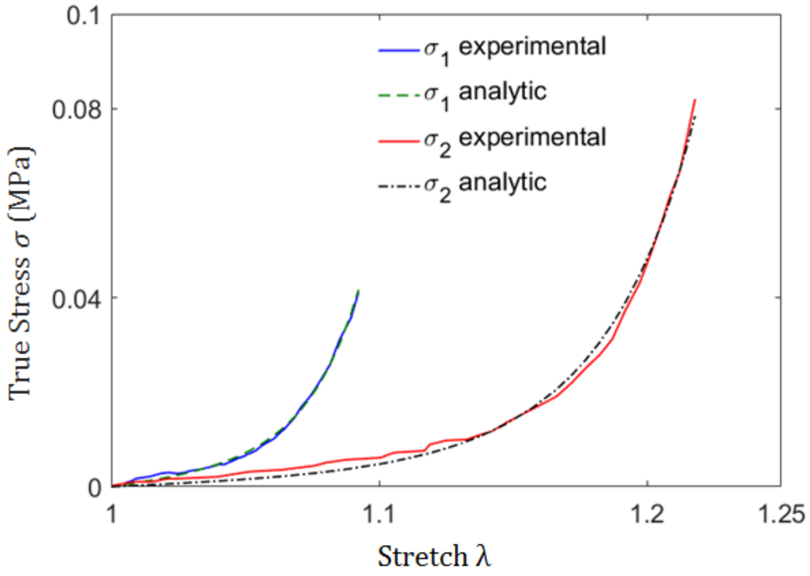
For both material (A) and (B), we used the developed identification strategy detailed in chapter §3 to identify the corresponding parameters X of Fung model. Tab.1 presents the obtained results. It should be underlined that several runs have demonstrated the reproducibility of the identification procedure.

Figure.2 details the corresponding mono-axial stress-stretch $\sigma_i(\lambda_i)$ analytical curves (for longitudinal direction i = 1 and transverse one i = 2), deduced from Fung model using our identified parameters depicted in Tab.1 for the two materials. The experimental curves obtained from [9] for material (A) and from our experiments for material (B) are also plotted.

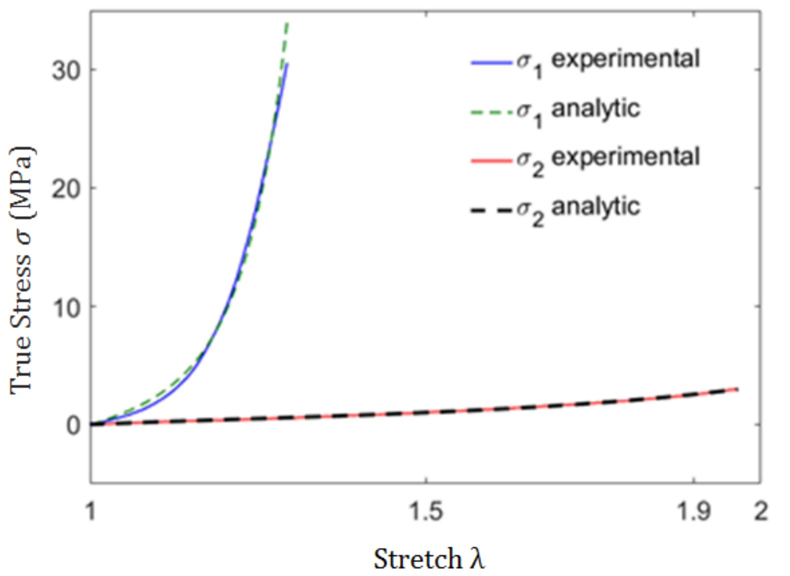
As can be seen, for both the adventitial layer material and for the woven tissue, the analytical curves deduced from parameters of Tab.1 appear to be in very good agreement with the experimental values. The developed identification strategy thus leads to realistic convex hyperelastic behavior for the two materials.

Material	C	B_{11}	B_{22}	B_{33}	B_{23}	B_{13}	B_{12}
Adventitial layer (A)	0.0004	165.857	47.028	135.448	46.747	0.799	14.587
Woven tissue (B)	1.64	11.787	0.0681	1.6412	0.0638	1.6411	0.0555

Table 1: Coefficients of Fung model



(A)



(B)

Figure 2: Experimental vs analytical results for adventitial layer (A) and woven material (B)

5 Prospects

In this study, a specific direct identification strategy, that allows taking into account additional convexity conditions, has been developed to identify Fung hyperelastic-anisotropic parameters. Comparisons with experimental data, for a biological soft tissue and a non-biological woven material, have demonstrated the usefulness efficiency of this approach.

This validation of the Fung identification procedure then allows us to work on the next step of the study: it will now focus on the FE implementation of Fung model, using the identified rheological parameters, to analyze numerically complex bi-axial tensile load cases for the woven tissue.

Références

[1] H. Abdul-Hameed, T. Messager, F. Zaïri, M. Naït-Abdelaziz, Large-strain viscoelastic-viscoplastic constitutive modeling of semi-crystalline polymers and model identification by deterministic/evolutionary approach, Computational Materials Science 90 (2014) 241-252

[2] Y.C. Fung, K. Fronek, P. Patitucci, Pseudoelasticity of arteries and the choice of its mathematical expression, American Journal of Physiology-Heart and Circulatory Physiology 237 (1979) 620-631

[3] M. T. Duong, Hyperelastic Modeling and Soft-Tissue Growth Integrated with the Smoothed Finite Element Method-SFEM, PhD, RWTH Aachen University, 2015

[4] G. A. Holzapfel, Nonlinear Solid Mechanics, A Continuum Approach for Engineering, John Wiley & Sons, 2000

[5] R.N. Vaishnav, J. T. Young, J. S. Janicki, D. J. Patel, Nonlinear anisotropic elastic properties of the canine aorta, Biophysical Journal 12 (1972) 1008-1027

[6] J.E. Bischoff, E.M. Arruda, K. Grosh, Finite element simulations of orthotropic hyperelasticity, Finite Elements in Analysis and Design 38 (2002) 983-999

[7] J.D. Humphrey, Mechanics of Arterial Wall: Review and Directions, Critical Reviews in Biomedical Engineering 23, (1995) 1-162

[8] Z. Michalewicz, Genetic Algorithms + Data Structures = Evolution Programs, 3rd Ed. Springer Verlag, Berlin, 1999

[9] G.A. Holzapfel, G. Sommer, T.C. Gasser, P. Regitnig P, Determination of layer-specific mechanical properties of human coronary arteries with nonatherosclerotic intimal thickening and related constitutive modeling, American Journal of Physiology-Heart and Circulatory Physiology 289 (2005) 2048-2058