Local and global generic separating sets for 3D elasticity tensors

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Résumé

Assuming that one could measure the elasticity tensors of two materials, it is a natural question to ask, if one can decide by finitely many calculations, whether the two materials have identical elastic properties (are identical as elastic materials), in other words if the two elasticity tensors are related by a rotation. We say then that the two tensors are in the same orbit. Based on the fact that the algebra of invariant polynomials of a linear representation of the rotation group is finitely generated and separates the orbits, abstract invariant theory gives an affirmative answer to this question. Nevertheless, calculating explicitly a generating set for this invariant algebra can be an extremely difficult task. A minimal integrity basis of 297 polynomial invariants for the invariant algebra of the 3D elasticity tensor was obtained in 2017. It is also a separating set but of quite high cardinal!

Since most materials have no symmetry in practice (they are triclinic), their membership to higher-symmetry classes is just a convenient approximation of the reality. Therefore, the notion of separating set/functional basis can be weakened, in order to reduce its cardinal. To be more specific, the notion of weak separating set - also known as a weak functional basis - has been formulated by Boehler, Kirillov and Onat in 1994, in the sense that they separate only generic tensors (defined rigorously in present work, using Zariski topology). Boehler, Kirillov and Onat produced a weak separating set of 39 polynomial invariants for generic elasticity tensors.

By formulating slightly different genericity conditions, we produce a weak separating set of 21 polynomial invariants for the elasticity tensor. This result is our main theorem. Moreover, translating results on rational invariants of the binary form of degree 8 by Maeda (1990), we can shorten this number to 19, but the corresponding polynomial invariants are more complicated. We can also deduce a set of 18 rational invariants which separate generic elasticity tensors. Cardinal 18 is minimal as it is equal to the transcendence degree, which is the maximal number of algebraic independent elements in the fractional field of the invariant algebra (Brion, 1996), here of the elasticity tensor. We also produce a local weak separating set of 18 polynomial invariants.

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