Behavioural laws for dams

I. Fontana[a,b], **D. Di Pietro**[c], **K. Kazymyrenko**[d]

a. EDF Lab, Paris-Saclay, France, ilaria.fontana@edf.fr
b. Université de Montpellier (UM), Unité de recherche IMAG, ilaria.fontana@etu.umontpellier.fr
c. Université de Montpellier (UM), Unité de recherche IMAG, daniele.di-pietro@umontpellier.fr
d. EDF Lab, Paris-Saclay, France, kyrylo.kazymyrenko@edf.fr

Résumé

Les équipes d'ingénierie ont souvent recours aux simulations numériques par éléments finis afin d'identifier le comportement des ouvrages hydrauliques de grande dimension. Pour les barrage en béton, les modèles doivent être capable de représenter les non linéarités mécaniques des interfaces (rupture, glissement, contact) et de prendre en compte l'écoulement hydraulique à travers les fissures des ouvrages.
Notre objectif est l'amélioration de la compréhension des phénomènes physiques associés à la rupture (frottement, adhésion, dilatance et écrouissage) et l'enrichissement des lois de comportement existant en les développant dans le formalisme adéquat de matériaux standards généralisés. La piste principale de travail est liée à l'introduction de couplage endommagement-plasticité via le terme de l'écrouissage cinématique dans l'énergie.

Abstract

Engineering teams often use finite elements numerical simulations to study large hydraulic structures. In particular, the models for concrete dams must represent the non-linear behaviour of the interfaces (rupture, sliding, contact) and take into account hydraulic flow through the openings of the structure. Our goal is to improve the understanding of the mechanical phenomena associated with fracture (e.g. friction, adhesion, dilatancy, hardening) and to enrich the existing behavioural laws. In particular, all the developments are made within the formalism of Generalized Standard Materials [3], by introduction of the damage-plasticity coupling in the energy function.

Mots clefs: Lois mécaniques, ouvrages hydrauliques, rupture, couplage endommagement-plasticité
Key words: Mechanical laws, hydraulic structures, fracture, damage-plasticity coupling

1 Introduction

The incidents which have occurred on concrete dams (e.g. Bouzey 1895, Malpasset 1959) along with the recent results of auscultation have observed that the stability of hydraulic structures largely depends on the hydro-mechanic behaviour of the weakest zones of the valley-dam unit which can be found at the level of discontinuities in the structure and in the rock. These weak chains of the construction are mainly the faults of the support areas, the concrete zones in the dam, the concrete-rock contact areas of the foundation, and the joints between the dam's sections. The mechanical behaviour of these zones is strongly non-linear, but thanks to their well known localisation in the restricted area, the industrial studies on large structures are accessible, even so being still complicated. In addition to these difficulties, the method of construction of the dams, the technique of *clavage/sciage*[1]used and the multiple draining points of the structures make finite elements modelling complex.

The joints of dams can originate in different ways, and they generally can be represented as a rough discontinuity, filled with some material (e.g. clay, grout or rock elements for the cracks between the dam and the foundation). These joints display various important phenomena: friction, loss of tensile strength, elastic behaviour for very small displacements, progressive disappearance of the peak of the shear stress with a cyclic loading. The relevance of these phenomena depends on different physical parameters, such as the roughness level, the mean size of the asperities, the mechanical properties of the filling materials, Young modulus, Poisson ratio, the friction coefficient. As a consequence, in order to obtain a complete model for joints, a law with several parameters has to be introduced. These mechanical laws are then introduced in some computing codes which use finite elements. The idea is to model the interfaces with a particular type of finite elements called cohesive zone (or interface) elements which have a flat geometry [6]. Some simplified laws have been implemented in the Code_Aster software developed by EDF: cohesive fracture law (called JOINT_MECA_RUPT) and Coulomb friction law (called JOINT_MECA_FROT) [4]. Both of the laws introduce a single mechanical phenomenon (damage or plasticity) coupled to hydraulic flow through the interface openings. However, during industrial simulations on dams performed by Centre d'Ingénierie Hydraulique EDF it was found that considering plasticity and damage separately does not allow to recover fully the behaviour observed experimentally. Another problem often found with these two laws is related to its weak convergence. For these reasons, a goal of EDF is to improve these implementations.

2 Mechanical behaviour

In this section we describe the context and the assumptions under which the mechanical laws are developed: cohesive zone model, energy formalism, generalized standard materials.

The cohesive zone model (CZM) introduced by Barenblatt [1] and Dugdale [2], is used to describe the evolution of damage. This choice is motivated by the fact that, in the case of brittle fracture, it allows to avoid infinite stress at the crack tip introducing some cohesion forces. In our case, we consider the jump between the two surfaces of the joint, e.g. between the dam and the foundation. Moreover, the equilibrium displacement field $\mathbf{u}$ is obtained as the minimum of a suitable energy function ψ and a variational approach is used in order to find the solution. The derivative of ψ with respect to the jump of displacement $\boldsymbol{\delta} = (\delta_n, \delta_{t_1}, \delta_{t_2})^T$ is denoted by $\boldsymbol{\sigma} = (\sigma_n, \sigma_{t_1}, \sigma_{t_2})^T$ and represents the cohesion force on the lips of the crack. Here, the subscript n denotes the normal component whereas the subscripts t_1 and t_2 denote the tangential components. A mechanical behaviour law for joints is then a relation between the jump $\boldsymbol{\delta}$ and its energy dual variable, i.e., the stress vector $\boldsymbol{\sigma}$. In this law, the stiffness coefficients k_n, k_{t_1} and k_{t_2} are involved. They are not equivalent, since they represent two different phenomena:

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[1] *Clavage* is the reinforcement of the dam made with an injection of concrete in the space between two blocks. *Sciage* is the act of cutting a block of the dam submitted to a too high compression force.

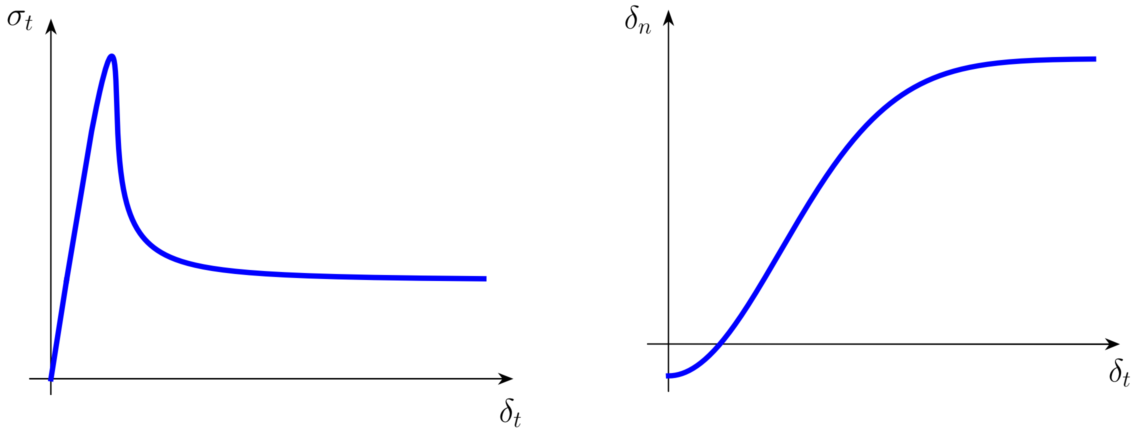

Figure 1: Behavioraul law in tangential loading for joints

the normal coefficient mostly depends on the stiffness of the filling material, whereas the tangential coefficients depend on the bending stiffness of the asperities. Moreover, it is possible to introduce other two state variables inside of the energy function: the vector of plastic displacement $\boldsymbol{p} = (p_n, p_{t_1}, p_{t_2})^T$ and the damage variable α. As for the jump $\boldsymbol{\delta}$, these two state variables are related to their energy dual variables which are the partial derivative of ψ with respect to $\boldsymbol{p}$ and α. These derivatives are denoted by $\boldsymbol{X}$ and Y respectively. This allows us to consider a wider range of behaviour laws, where evolution of damage and plasticity is governed by dissipation depending exclusively on $\boldsymbol{X}$ and Y.

Experimentally, the mechanical behaviour in tangential loading is shown in Figure 1 (for simplicity only one tangential component is considered). As regards the strain-stress relation (plot on the left) we can observe a softening behaviour, that occurs after a short elastic phase, due to the fact that the damage begins to act progressively. This reflects the fact that, until the filling material is not damaged or the asperities are not broken, the behaviour of the joint remains elastic. Moreover, there is a peak for the value of σ_t and in both of the plots we can observe a flat asymptotic behaviour. It is important to observe that different types of joints of dams can show different profiles of asperities (in particular, as regards their depth). As a consequence, we can obtain different behaviours increasing the tangential displacement. However, we can always recover the flat asymptotic behaviour, that is, the joint will slide with a specific friction coefficient asymptotically. In order to recover the behaviour of Figure 1 we consider both plasticity and damage in the energy expression and we assume that they evolve simultaneously (as in [5]).

In our model, we choose also the formalism of generalized standard materials [3] for which the reversibility domain is convex and the flow rule for plasticity follows the normality rule in the smooth points of its boundary. If the boundary of the domain is not smooth (i.e., there are points of the boundary in which the normal is not defined), this rule can be extended using the Hill maximal work principle [7, Section 5.3.3]. As first approach, we assume that the domain is determined by the Drucker–Prager criterion [5, Section 2.3]. An example of reversibility domain determined by the Drucker–Prager criterion is shown in Figure 2. This criterion is a natural consequence of the assumption that the sliding with friction follows the Coulomb law which depends only on two parameters, the friction coefficient $\mu \in (0, +\infty)$ and cohesion τ. The Coulomb law achieves the non-interpenetration condition of the interfaces (Signorini's condition) since it establishes a local relation between the tangential and normal stress during the sliding phase: $\|\boldsymbol{\sigma}_t\| = \tau - \mu\sigma_n$.

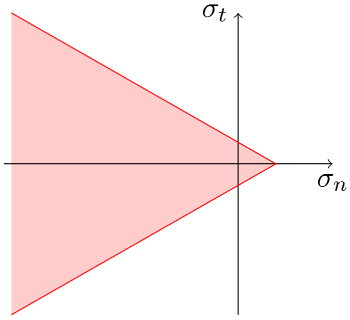

Figure 2: The convex domain

We have analysed first the following form for the energy function in the 2-dimensional case:

$$\psi(\boldsymbol{\delta}, \boldsymbol{p}, \alpha) = \frac{1}{2}k_n(\delta_n - p_n)^2 + \frac{1}{2}k_t(\delta_t - p_t)^2 + \frac{1}{2}a_{11}(\alpha)p_n^2 + \frac{1}{2}a_{22}(\alpha)p_t^2 + D_1\alpha \tag{1}$$

where $k_n, k_t > 0$, $a_{11}(\alpha) = R_1\dfrac{(1-\alpha)^3}{\sqrt{\alpha}}$, $a_{22}(\alpha) = R_2\dfrac{(1-\alpha)^3}{\sqrt{\alpha}}$ and $D_1 > 0$ is the maximal damage dissipated energy. With these hypotheses, we have obtained the behaviour shown in Figure 3: we have recovered the main features of behaviour of the tangential stress σ_t (initial elastic phase, peak and asymptotic flatness) whereas the normal displacement δ_n diverges asymptotically. Therefore, this choice does not allow to describe both of the behaviours of Figure 1 and we have to consider new behavioural laws. In particular, we want to investigate two possible ways: the case of hyperelasticity and the case in which the yield surface of Figure 2 depends on the damage α.

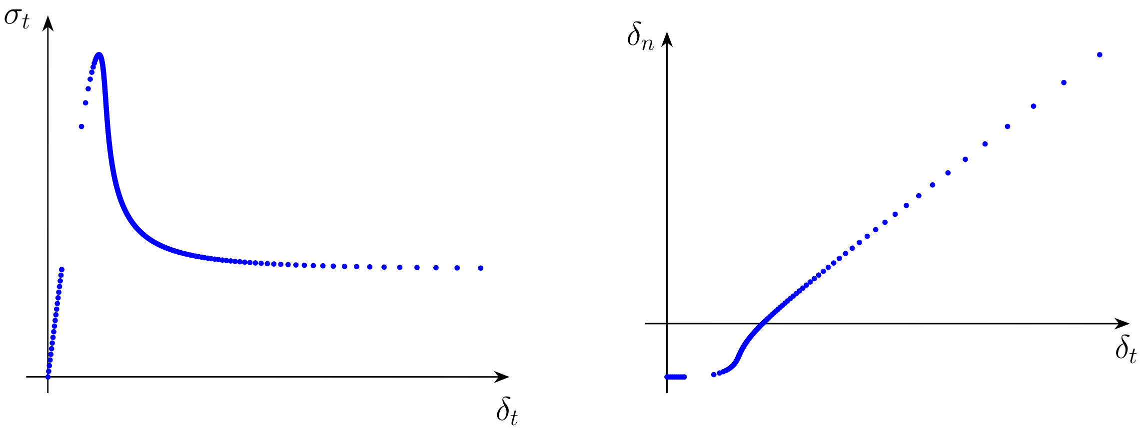

Figure 3: Computed behavioural law in tangential loading

3 Conclusions

Motivated by the results obtained in [5] coupling plasticity and damage, the analysis of a particular form of free energy function with Drucker–Prager criterion gives promising results reproducing some relevant mechanical behaviour of interfaces in concrete dams. The goal of the future presentation is to explain these ideas and propose modifications that can be applied in order to recover both of the behaviours (sliding frictional pic and dilatance saturation) of Figure 1.

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