Sudden transition from non-swirling to swirling axisymmetric turbulence

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Résumé :

Des simulations numériques directes d'un écoulement turbulent strictement axisymétrique et stationnaire sont réalisées dans un domaine cylindrique. La méthode de forçage permet de contrôler séparément l'injection d'énergie dans les directions toroïdale (azimutale) et poloïdale (radiale et axiale). Nous étudions la transition d'un état sans 'swirl' à l'état avec, en faisant varier le rapport entre les coefficients de forçage toroïdal et poloïdal C_t/C_p*. Le rapport entre les composantes toroïdale et poloïdale de l'énergie, en fonction de* C_t/C_p *montre la présence d'un phénomène de bifurcation. La transition se produit lorsque le ratio de forçage est proche de* 1*. Nous proposons ensuite un modèle basé sur les équations régissant les composantes toroïdales et poloïdales de l'énergie. Ce modèle montre un excellent accord avec les résultats de la simulation numérique directe.*

Abstract :

Direct numerical simulations of strictly axisymmetric and stationary turbulence are carried out in a cylindrical domain. The forcing method allows to control separately the energy injection in the toroidal (azimuthal) and poloidal (radial and axial) directions. We investigate the transition from non-swirling to swirling states by varying the ratio between the toroidal and poloidal forcing coefficients C_t/C_p*. Plotting the swirl indicator, defined as the ratio between the toroidal and poloidal energy components, as a function of* C_t/C_p *shows evidence of a bifurcation phenomenon. The transition occurs when the forcing ratio is close to* 1*. We then propose a model based on the equations governing the toroidal and poloidal energy components. This model displays an excellent agreement with the results of the direct numerical simulation.*

Keywords : Axisymmetric turbulence, transition, direct numerical simulation, turbulence modelling

1 Introduction

Recently, transitions between different turbulent states have received a considerable amount of interest. In particular the transition between two-dimensional and three-dimensional turbulent flow has been investigated in thin fluid layers, both by experiments [1, 2] and numerical simulations [3, 4]. A review of such turbulent transitions is given in [5].

A different transition between turbulent states is the transition between 2- and 3-component flows. A typical example of a three dimensional-two component (3D2C) flow is strongly stratified turbulence, where the movements are almost entirely confined to the plane perpendicular to an imposed density gradient, but these two velocity components vary strongly in the three directions. Examples of nearly two-dimensional three-component (2D3C) flows are fastly rotating turbulence and turbulence of a conducting fluid in the presence of a strong magnetic field. These flows have three velocity components but are almost invariant along the direction of the axis of rotation or magnetic field, respectively.

In the present investigation we will numerically study such a kind of transition in axisymmetric turbulence, another example of 2D3C flow. Strictly axisymmetric turbulence, *i.e.* turbulence governed by the Navier-Stokes equations modified such that the flow is invariant in the azimuthal direction, is a system intermediate between two- and three-dimensional turbulence. Recent numerical simulations showed in particular that this system allows for an inverse energy cascade, responsible for the generation of large scale coherent structures, and a direct helicity cascade towards small scales [6, 7]. As predicted from theoretical works using statistical mechanics tools [8, 9], different behaviors were obtained for swirling and for non-swirling flows. Let us recall here that in the latter the toroidal movements are negligible with respect to poloidal ones (see Fig. 1), while in the former the toroidal velocity is of the same order as its poloidal components. We investigate here the transition from the non-swirling (two dimensional-two components, 2D2C) to the swirling regime (two dimensional-three components, 2D3C), focusing on the bifurcation between the two states.

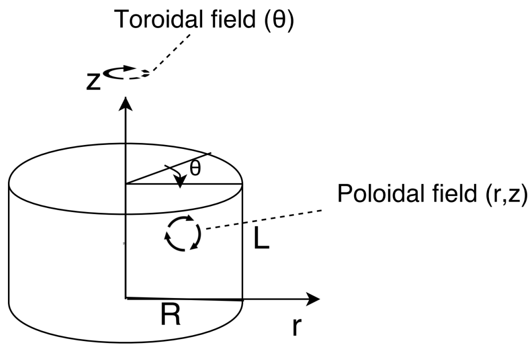

FIGURE 1 – Computational box and definition of the toroidal/poloidal components of velocity.

2 Numerical method and definitions

We begin with the incompressible axisymmetric Navier-Stokes equations written in cylindrical coordinates :

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0, \tag{1a}$$

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} \right) + f_r, \tag{1b}$$

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = \nu \left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} \right) + f_\theta, \tag{1c}$$

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} \right) + f_z, \tag{1d}$$

where u_r, u_θ and u_z are the velocity components in the radial, azimuthal and axial directions respectively, p is the pressure field, ρ is the fluid density and ν its kinematic viscosity, and $\mathbf{f}$ is a forcing term. This forcing is such that energy is injected into the system within a certain wavenumber band. We will hereafter distinguish the poloidal velocity field (u_r, u_z) from the toroidal one u_θ (see Fig. 1). The parameters C_p and C_t respectively denote the amplitudes of the forcing in the poloidal and toroidal directions.

The system (1a-1d) is integrated numerically in a cylindrical domain by using a fully spectral method based on an expansion of the velocity field using Chandrasekhar-Kendall helical eigenfunctions of the curl [10]. More details on this method can be found in [6, 7].

The total kinetic energy E can be written as the sum of the poloidal and toroidal energy components, respectively defined as :

$$\begin{aligned} E_\mathrm{p} &\equiv \frac{1}{2}\langle u_r^2 + u_z^2 \rangle, \\ E_\mathrm{t} &\equiv \frac{1}{2}\langle u_\theta^2 \rangle, \\ E &= E_\mathrm{t} + E_\mathrm{p}, \end{aligned} \tag{2}$$

where $\langle ... \rangle$ denotes the volume average over the cylindrical domain.

3 Numerical results

The simulations were carried out in a domain of radius $R = \pi$ and of height $L = 1.7\pi$. The kinematic viscosity was set to $\nu = 0.01$, and the forcing parameters C_p and C_t were varied within the interval [0.02,0.072]. These settings result in a range of large scale Reynolds number $\mathrm{Re} \equiv \sqrt{2E/3}R/\nu \in [730, 1280]$.

We investigate the transition from non-swirling to swirling regimes by measuring a swirl indicator γ, defined as the ratio between the toroidal and poloidal energy components ($\gamma = E_t/E_p$), as a function of the ratio of the toroidal/poloidal forcing coefficients, C_t/C_p. The pure non-swirling case then corresponds to a swirl indicator $\gamma = 0$. Our results are gathered in Fig. 2. The numerical data clearly shows the presence of a bifurcation from the non-swirling (2D2C, 2 dimensions-2 components) to the swirling (2D3C, 2 dimensions-3 components) regimes. The transition occurs for a forcing ratio close to 1. We propose in the next section a model based on the dynamical equations of the toroidal and poloidal energy components in order to interpret this sudden change of flow regime.

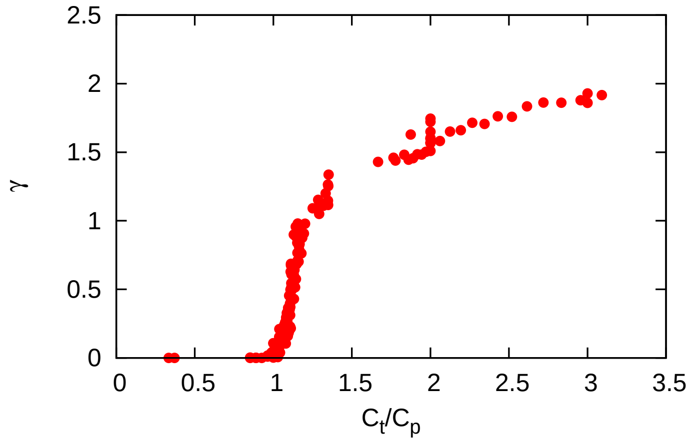

FIGURE 2 – Swirl indicator γ, defined as the ratio between the toroidal and poloidal energy components, plotted as a function of the ratio between the toroidal and poloidal forcing coefficients (direct numerical simulation data).

4 Model

Exact equations for the poloidal and energy components can be derived from the axisymmetric Navier-Stokes equations (1a-1d). These equations formally read :

$$\frac{\partial E_{\mathrm{p}}}{\partial t} = T_{\text{t-p}} + F_{\mathrm{p}} - \varepsilon_{\mathrm{p}}, \tag{3}$$
$$\frac{\partial E_{\mathrm{t}}}{\partial t} = -T_{\text{t-p}} + F_{\mathrm{t}} - \varepsilon_{\mathrm{t}}, \tag{4}$$

where F_{p} and F_{t} are the forcing contributions, ε_{p} and ε_{t} are energy dissipation rates, and $T_{\text{t-p}}$ represents the energy transfer from the toroidal component to the poloidal one :

$$T_{\text{t-p}} = \langle \frac{u_r u_\theta^2}{r} \rangle. \tag{5}$$

Our aim is to approximate each term on the right-hand sides of Eq. (3, 4). Assuming that $\langle u_r^2 \rangle \approx \langle u_r^2 + u_z^2 \rangle / 2 = E_{\mathrm{p}}$ and using dimensional estimates based on standard turbulence modelling, one gets for the transfer term :

$$T_{\text{t-p}} = \langle \frac{u_\theta^2 u_r}{r} \rangle \sim \mathcal{T}_1 \left[\frac{4E_{\mathrm{t}}^2}{R^2} - 4\frac{E_{\mathrm{t}} E_{\mathrm{p}}}{R^2} \right], \tag{6}$$

where $\mathcal{T}_1$ is a characteristic time scale which can be estimated as $\mathcal{T}_1 = \zeta \frac{R}{\sqrt{E_{\mathrm{p}}}}$, where ζ is a dimensionless coefficient.

The two forcing terms are modelled as :

$$F_{\mathrm{p}} \sim 2C_{\mathrm{p}} A_{\mathrm{p}} E_{\mathrm{p}}, \tag{7}$$
$$F_{\mathrm{t}} \sim 2C_{\mathrm{t}} A_{\mathrm{t}} E_{\mathrm{t}}, \tag{8}$$

where C_{p} and C_{t} are dimensionless parameters of the model.

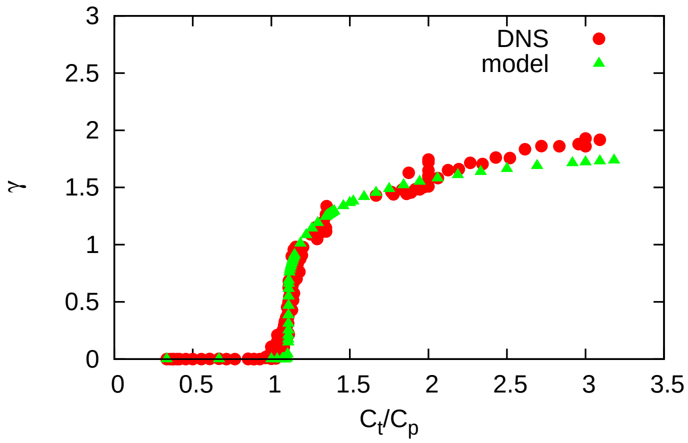

FIGURE 3 – Swirl indicator γ, defined as the ratio between the toroidal and poloidal energy components, plotted as a function of the ratio between the toroidal and poloidal forcing coefficients (direct numerical simulation and model).

The dissipation terms are modeled as,

$$\varepsilon_{\rm p} \sim \delta_{\rm p}\frac{U_{\rm p}^2}{\mathcal{T}_2} \sim \delta_{\rm p}\frac{E_{\rm p}E^{1/2}}{R}, \tag{9}$$

$$\varepsilon_{\rm t} \sim \delta_{\rm t}\frac{U_{\rm t}^2}{\mathcal{T}_2} \sim \delta_{\rm t}\frac{E_{\rm t}E^{1/2}}{R}, \tag{10}$$

where $\delta_{\rm p}$ and $\delta_{\rm t}$ are dimensionless dissipation coefficients, $U_{\rm p} = E_{\rm p}^{1/2}$ and $U_{\rm t} = E_{\rm t}^{1/2}$ are the characteristic velocities in the poloidal and toroidal directions, and $\mathcal{T}_2 = R/E^{1/2}$ is a characteristic time scale. This model implies that the energy dissipation rates are determined by the large scales.

Overall, the system (3, 4) is therefore modelled as :

$$\frac{{\rm d}E_{\rm p}}{{\rm d}t} = \frac{4\zeta E_{\rm t}\sqrt{E_{\rm p}}(\gamma^2-1)}{R} + 2C_{\rm p}A_{\rm p}E_{\rm p} - \delta_{\rm p}\frac{E_{\rm p}E^{1/2}}{R}, \tag{11}$$

$$\frac{{\rm d}E_{\rm t}}{{\rm d}t} = -\frac{4\zeta E_{\rm t}\sqrt{E_{\rm p}}(\gamma^2-1)}{R} + 2C_{\rm t}A_{\rm t}E_{\rm t} - \delta_{\rm t}\frac{E_{\rm t}E^{1/2}}{R}. \tag{12}$$

The values of the model parameters ζ, $A_{\rm p}$, $A_{\rm t}$, $\delta_{\rm p}$ and $\delta_{\rm t}$ have been determined by fitting the numerical data. The solutions of the model with these values of the parameters are compared to those obtained by direct numerical simulation of the Navier-Stokes equations, as shown in Fig.3. Both data sets exhibit an excellent agreement. The model reproduces in particular the bifurcation observed for $C_{\rm t}/C_{\rm p} \approx 1$.

5 Conclusions and perspectives

We have provided evidence of a transition from non-swirling to swirling regimes of axisymmetric turbulence using direct numerical simulation. Plotting the swirl indicator γ as a function of the ratio between the toroidal and poloidal forcing coefficients, a bifurcation between the non-swirling ($\gamma = 0$) and the swirling ($\gamma = \mathcal{O}(1)$) regimes was clearly observed. This transition occurs for a forcing ratio close to 1. We then proposed a model based on the dynamical equations of the toroidal and poloidal energy components. The values of this model parameters were evaluated by fitting the DNS results. The model thus integrated displays an excellent agreement with the results of direct numerical simulation.

The model may be extended to characterize the dynamics of three-dimensional (non-axisymmetric) turbulence in cylindrical coordinates, which would need the modeling of the pressure strain correlations. Such a model could indicate how different the observed dynamics are from the behaviour of three-dimensional turbulence.

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