# Computational framework of repeated load CBR (Californian Bearing Ratio) testing

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## Abstract:

*Repeated Load Californian Bearing Ratio test (RL-CBR) is a geotechnical laboratory test similar to the well-known CBR test with cyclic loading. The RL-CBR test was developed at the Netherlands University to study unbound granular materials resilient behaviour by estimating their equivalent modulus. In this case, three model parameters must be determined by Finite Element Method (FEM) and non-linear multidimensional regression between equivalent modulus as a response variable and Poisson's ratio ν, mean stress under plunger and recoverable plunger penetration as explanatory variables. The purpose of this study is to show how FEM inputs affect the obtained model parameters. The estimation procedure consists of the simulation of RL-CBR test varying elastic materials characteristics and plunger penetrations. For each combination, nodal reactions under plunger are captured and mean stress under plunger is calculated. A non-linear multidimensional regression allows deriving equivalent modulus equation by the Least-Squares Method which consists of minimizing the quadratic error between real and estimated modulus. In practice, these model parameters depend also on mould characteristics and specimen-mould contact nature. In this paper, the influence of these parameters on the model's parameters' values is studied to determine the adequate set of parameters for carrying out an accurate estimation of equivalent modulus' parameters.*

**Keywords: granular material, Repeated Load CBR, Equivalent Modulus, stiffness, Finite Element Method, Least-Squares Method**

# 1. Introduction

Stiffness modulus of soils and unbound granular materials (UGM) is a key input characteristic in Mechanistic-Empirical (M-E) design process of flexible pavements. However, its evaluation is a challenge in road engineering. Many correlations allow for estimation of the soil's modulus based on the Californian Bearing Ratio index (CBR) used worldwide. However, the recent studies [1] have proved using finite element elastoplastic simulation that in addition to the material's Young's modulus, the CBR index depends also on other parameters such as yield stresses in compression and the compressibility index, this last parameter was, sometimes, more important than Young's modulus. The RLT test (Repeated Load Triaxial) is the most accepted and used test in research laboratories to study the resilient and permanent behaviour of these materials. However, the set-up of this test is technically complex and equipment is expensive. Hence, its use in road laboratories is not affordable, especially in developing countries [2,3]. To overcome this challenge and in the absence of triaxial test results  French standard of M-E road pavement design [4] adopts modulus based on empirical standard UGM characterization. In the RLT test the specimens are loaded by a confining stress $\sigma_3$ in axial and radial directions, and an axial deviator stress $q = \sigma_1 - \sigma_3$ where $\sigma_1$ is the total axial stress. To simulate traffic load repetition $q$ is usually a periodic stress, whereas, $\sigma_3$ may be periodic for Variable Confining Pressure test variant (VCP) or constant for Constant Confining Pressure test variant (CCP). For RLT test, specimen has a diameter of 160 mm or 300 mm and a height of 320 mm or 600 mm respectively. RL-CBR test is validated based on resilient modulus derived from large CCP triaxial test (300×600 specimen), and those derived from RL-CBR with strain gauges, using a steel mould with an internal diameter of 250 mm, a height of 200 mm and a wall thickness of 14,5 mm [2,5]. In this study we consider the same mould with 8 mm thickness.

# 2. Characterization of resilient behaviour of granular materials using RL-CBR

Experimental characterization of granular materials is a large research field. Their mechanical behaviour depends on large number of parameters; many experimental and theoretical models are available in scientific literature for describing resilient behaviour. These tasks are discussed in detail in state of art papers [6–8]. Many tests are used to characterize the resilient behaviour of unbound granular materials. When the RLT test is used, resilient modulus is evaluated by the equation (1) for VCP variant and by the equation (2) for CCP variant according to the European standard [9]. Other tests [10] can be used to study the resilient behaviour of granular materials.

$$M_r = \frac{\sigma_{r1}^2 + \sigma_{r1}\sigma_{r3} - 2\sigma_{r3}^2}{\sigma_{r1}\epsilon_{r1} + \sigma_{r1}\epsilon_{r3} - 2\sigma_{r3}\epsilon_{r3}} \tag{1}$$

$$M_r = \frac{\sigma_{r1}}{\epsilon_{r1}} \tag{2}$$

where:

$M_r$ : Resilient modulus (MPa)
$\sigma_{r1}$ : Resilient axial stress ($\sigma_{r1} = \sigma_{r1max} - \sigma_{r1min}$ ) (MPa)
$\sigma_{r3}$ : Resilient confining stress ($\sigma_{r3} = \sigma_{r3max} - \sigma_{r3min}$ ) (MPa)
$\epsilon_{r1},\ \epsilon_{r3}$ : Resilient axial and radial strain ($\epsilon_{ri} = \epsilon_{rimax} - \epsilon_{rimin}$ i=1,3) (-)

In the framework of RL-CBR test the stiffness of UGM is evaluated by resilient modulus designed as equivalent modulus by Araya [2]  the used expression is inspired from the Boussinesq equation (3) in the case of elastic isotropic semi-infinite solid loaded by a circular plunger. For RL-CBR, a mould with finite dimensions is used, then Araya [2] suggests modifying the expression (3) by using another (4) introducing three model parameters determined by Least-Squares Method (LSM) (see 3.2.2. ) applied on RL-CBR numerical data analysis. Opiyo, in his MSc thesis [11], was the first researcher to adopt this method, he had considered the standard CBR mould that can be used for soils with maximum grain size less than 20 mm.  The expression (4) is used in the case of RL-CBR without strain gauges. In the other case, Araya [5] developed a sophisticated system of transfer functions that offers estimation of Poisson's ratio and equivalent modulus.

$$u = \frac{f(1-\nu^2)\sigma_p \frac{d}{2}}{E} \Leftrightarrow E = \frac{f(1-\nu^2)\sigma_p \frac{d}{2}}{u} \tag{3}$$

$$E_{eq} = \frac{k_1\left(1-\nu^{k_2}\right)\sigma_p \frac{d}{2}}{u^{k_3}} \tag{4}$$

where:

$u$ : Resilient plunger penetration (mm)

$f$ : Constant factor: = 2 for flexible plunger, $\pi/2$ for stiff plunger

$\sigma_p$ : Mean stress under plunger (MPa)

$d$ : Plunger diameter (mm)

$E$ : Elastic modulus (MPa)

$E_{eq}$ : Equivalent modulus (MPa)

$k_1, k_2, k_3$ : Model parameters (.)

Araya [2] calls the derived resilient modulus “Equivalent modulus”, while Molenaar [12] calls it “effective modulus”. In the RL-CBR set-up, the state of stress throughout specimen is not uniform; therefore, the resilient modulus may vary throughout it, then, the equivalent or effective modulus is a bulk measurement of the specimen stiffness rather than an intrinsic material’s characteristic.

# 3. Materials and Methods

## 3.1. Materials properties and loading conditions

In this work a linear elastic material property is considered for the granular materials. A large quality ranges of UGM were studied by varying the Young’s modulus value from 25 MPa to 1000 MPa by a variable step and Poisson ratio, ν, from 0,15 to 0,45 by a step of 0,10. Plunger penetrations used in this study vary from 0,1 mm to 3 mm. The first set of parameters consists of combining Young modulus from 25 MPa to 1000 by a step of 25 MPa (40 values), Poisson ratio with the list values above (4 values) and plunger penetration (u) from 0,1 mm to 3 mm by a steep of 0,1 mm (30 values). In total, we have 4800 simulations of the RL-CBR test which would take a large time calculation. As a consequence, we need to reduce the number of simulations by choosing the ones that offer accurate estimation of the model parameters. Firstly, we will try to reduce the list of Young's moduli. Subsequently, we will decrease the plunger’s penetrations list length. Table 1 summarizes parameters’ set used to reduce the length of Young’s moduli list; only the step between two consecutive modulus’ values was changed. We will see in section 4. that set 5 gives accurate parameters’ values compared to those obtained in the first set while the simulations’ number is reduced from 4800 to 600. Table 2 summarizes the sets of parameters tested to reduce plunger’s penetrations list. Set 5 is the initial set for this stage of the study. Then, step between consecutive penetrations is varied from 0,1 mm to 1 mm .

Table 1. Sets of parameters tested to reduce the list of Young's modulus E

| **Set** | ***E* step (MPa)** | ***ν* step (-)** | ***u* step (mm)** | **Number of simulations** |
|---|---|---|---|---|
| Set 1 | 25 | 0,1 | 0,1 | 4800 |
| Set 2 | 50 | 0,1 | 0,1 | 2400 |
| Set 3 | 100 | 0,1 | 0,1 | 1200 |
| Set 4 | 125 | 0,1 | 0,1 | 960 |
| Set 5 | 200 | 0,1 | 0,1 | 600 |

| | | | | |
|---|---|---|---|---|
| Set 6 | 250 | 0,1 | 0,1 | 480 |
| Set 7 | 500 | 0,1 | 0,1 | 240 |

Table 2. Sets of parameters tested to reduce the list of plunger penetrations u

| **Set** | ***E* step (MPa)** | **ν step (-)** | **$u$ step (mm)** | **Number of simulations** |
|---|---|---|---|---|
| Set 5 | 200 | 0,1 | 0,1 | 600 |
| Set 8 | 200 | 0,1 | 0,2 | 300 |
| Set 9 | 200 | 0,1 | 0,3 | 200 |
| Set 10 | 200 | 0,1 | 0,5 | 120 |
| Set 11 | 200 | 0,1 | 0,6 | 100 |
| Set 12 | 200 | 0,1 | 1 | 60 |

## 3.2. Methods

### 3.2.1. Finite element analysis of RL-CBR test

Finite element simulation of Repeated Load CBR test is performed with the commercial Abaqus Software. As there is a symmetric axis for geometry, loading and boundary conditions of the RL-CBR set up, an axisymmetric approach is used in the modelling process. A linear elastic material property is assumed for the steel mould and the granular material with an elastic modulus of 210 GPa and a Poisson's ratio of 0.3 using the variable elastic characteristics presented in section 3 for granular material. As for standard CBR test, RL-CBR is a strain-controlled test with a uniform displacement of the plunger through material specimen. Assuming the plunger to be a rigid body, only its effect is modelled: an imposed displacement that takes several values $u_i^k$ $(0 < u_i^k \leq 3\ mm)$ for each elastic parameters' combination $(E_i^k; \nu_i^k)$ where k is the set index. For the normal contact property, the hard pressure-overclosure defined in ABAQUS is adopted between the two parts of the model. For the tangential interaction, frictionless formulation between specimen and mould was chosen. These considerations mean that neither penetration nor friction between specimen and mould will take place when local contact between specimen and mould is established. The use of frictionless contact assumes that the internal mould surface is very smooth and a demoulding oil is used in test preparation to replicate the smoothness. The Fig. 1 shows the axisymmetric model used in finite element analysis of RL-CBR test. The number of elements and nodes in the mesh are 780 and 2236 respectively. CAX8R, an 8-node biquadratic axisymmetric quadrilateral reduced integration elements type is used for specimen mesh, this type of elements is used because it offers an accurate simulation of a three-dimensional problem using plane axisymmetric model. For the mould meshing only CAX4R element type is used because a higher analysis accuracy is not needed for the mould part.

### 3.2.2. Least-Squares Method

After conducting a set of RL-CBR numerical analysis (section 3.1. ), the data is organized as illustrated in Table 3. In the analysis process, the Young modulus $E_i$, Poisson's ratio $\nu_i$ and plunger penetration $u_i$ are the input parameters and mean stress under plunger $\sigma_{pi}$ is the output parameter. In the regression analysis, with respect to model (4), elastic modulus is considered as the response variable and other parameters as explanatory variable. However, the objective is to derive an equivalent modulus expression to be used in experimental characterization of soils and granular materials using RL-CBR where the stiffness is researched. Resilient plunger penetration and mean resilient stress are measured during the test, the Poisson's ratio is estimated and generally a 0,35 Poisson's ratio is considered for soil and UGM [4].

For a set of observation, the object is to find the three model parameters: $k_1$, $k_2$ and $k_3$ that allow the estimation of response variable knowing explanatory variables. These parameters should give an equivalent modulus as close as possible for the initial elastic modulus for each variables' combination. There are three explanatory variables and the chosen model (4) is non-linear and the non-linear multivariate regression problem is treated her by LSM. The principle of LSM consists of researching model parameters minimizing the Sum of Squared Deviations (SDS) defined for our case by expression (5). The General Reduced Gradient method is used to solve this non-linear optimization problem summarized in equation (6).

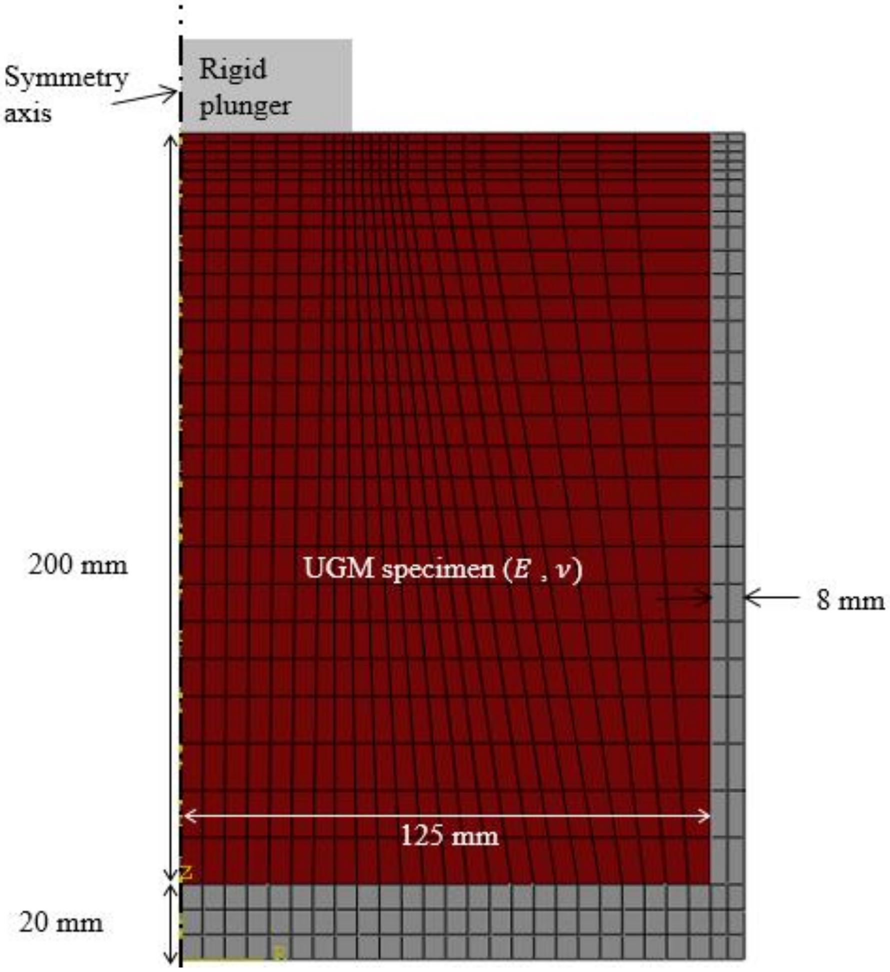

Fig. 1 Axisymmetric finite elements model of RL-CBR test

Table 3. Table of variables for a set of analysis

| Analysis number | Response E (MPa) | Explanatory variables | | |
|---|---|---|---|---|
| 1 | $E_1$ | $\nu_1$ | $u_1$ | $\sigma_{p1}$ |
| 2 | $E_2$ | $\nu_2$ | $u_2$ | $\sigma_{p2}$ |
| … | … | … | … | … |
| $i$ | $E_i$ | $\nu_i$ | $u_i$ | $\sigma_{pi}$ |
| … | … | … | … | … |
| n | $E_n$ | $\nu_n$ | $u_n$ | $\sigma_{pn}$ |

$$SDS(k_1;\ k_2;\ k_3) = \sum_{i=1}^{i=n} \varepsilon_i{}^2 = \sum_{i=1}^{i=n} (E_i - E_{eqi})^2 = \sum_{i=1}^{i=n} \left( E_i - \frac{k_1\left(1 - \nu_i^{k_2}\right)\sigma_{pi}\frac{d}{2}}{u_i^{k_3}} \right)^2 \quad (5)$$

where:

| | | |
|---|---|---|
| $SDS(k_1;\ k_2;\ k_3)$ | : | Sum of squared deviations |
| $\varepsilon_i = E_i - E_{eqi}$ | : | Deviation between real and estimated response for i^th^ data line |
| $n$ | : | Number of data line |
| $E_i$ | : | Real response (elastic modulus) |
| $E_{eqi}$ | : | Estimated response (Equivalent modulus) |

Non-linear optimization problem : $$\begin{cases} Minimize\ S(k_1;\ k_2;\ k_3) \\ \text{constraints } 0 \le k_i\ ;\ i = 1, 2, 3 \end{cases} \quad (6)$$

# 4. Results and Discussion

Parameters of model (4) were determined for each data sets presented in Table 1 and Table 2. Results of simulations used to reduce the length of the Young's modulus list are summarized in Table 4. For each set, this table presents also the sum of squared deviation SDS defined in equation (5) and the correlation coefficient R. Table 4 shows that the first parameter was decreased by increasing the step between consecutive values of Young's modulus from 1,653 for a step of 25 MPa to 1,641 for the step of 500 MPa, let a reduction of 0,7%. But the second parameter was increased by 2% where the third parameter remained invariant for the seven sets. With respect to the correlation coefficient, all correlations seem to be good except that of 5th set where the correlation coefficient was equal to 0,995. However, R values presented in Table 4 can't be used to compare accuracy of solutions because of the differences between samples sizes. To do this comparison, SDS' values are calculated for each solution with respect to the finite elements simulations' results of the large sample (i.e. set 1). The Fig. 2 presents the variation of SDS depending on the set used to estimate model parameters. It seems that SDS is minimal when parameters resulting from set 1 are used which seems normal, because these parameters are estimated using the LSM for the same set. Then, SDS increases by increasing the Young's modulus step (set 2 to set 7). However, for parameters obtained from set 7 relative deviation from the first SDS has reached the maximum, about 1,3% greater than the first SDS. By comparing the relative deviations related to the use of each set with respect to the set 1, results show that set 5 offers an accurate estimation of model parameters. In this case, a 200 MPa Young's modulus step is used for estimating parameters of model (4) instead of 25 MPa steps used in set 1 without decreasing model estimation accuracy. This means that we divide the length of Young's modulus list by 8. This decrease will optimize analysis time and facilitate the estimation of model parameters for other moulds geometry and finite element models.

Table 4. Model parameters for sets 1 to 7 used to reduce the list length of Young's modulus E

| **Set** | **K1** | **K2** | **K3** | **Correlation coefficient R** |
|---|---|---|---|---|
| Set 1 | 1,653 | 0,978 | 1,001 | 0,998 |
| Set 2 | 1,652 | 0,979 | 1,001 | 0,998 |
| Set 3 | 1,651 | 0,982 | 1,001 | 0,997 |
| Set 4 | 1,650 | 0,983 | 1,001 | 0,997 |
| Set 5 | 1,648 | 0,986 | 1,001 | 0,997 |
| Set 6 | 1,647 | 0,988 | 1,001 | 0,997 |
| Set 7 | 1,641 | 0,998 | 1,001 | 0,995 |

The length list of Young’s moduli has gone from 40 values to 5 values keeping estimation accuracy in the same level. The same approach is used to reduce the length of plunger penetrations list. In this level, the starting set is set 5 chosen above, it’s noted for this set, modulus step is equal to 200 MPa and plunger penetration step is equal to 0,1 mm. For subsequent sets this step is increased from 0,1 mm to 1 mm, Table 2 presents adopted plunger penetration steps for each set.

Table 5 summarizes obtained parameters for this part of the study. It is noted that parameter’s values don’t change much, and, for the last four sets, values didn’t change. These results mean that reducing length of plunger penetration’s list doesn’t influence clearly model’s parameters values which can be explained by the chosen contact conditions between mould and specimen that make the problem linear, then, the ratio of mean stress $\sigma_{pi}$ by plunger penetration $u_i$was seen to be constant for a given specimen. The Fig. 3 shows the constancy of SDS evaluated for the 1st set when obtained parameters from sets 5, 8 to 12 are used. Only parameters resulting of set 8 gave the least accuracy estimation, for other case, the same accuracy is maintained.

To choose the set that makes the best compromise between the model accuracy and time consumption for calculation we should consider the non-linear character of the model. Accordingly, the lengths list of all parameter must be at least 3 which is the case of set 12 with 3 values of plunger penetrations: 1, 2 and 3 mm. To gain accuracy for other cases of simulation considering analysis conditions, the set 11 is chosen.

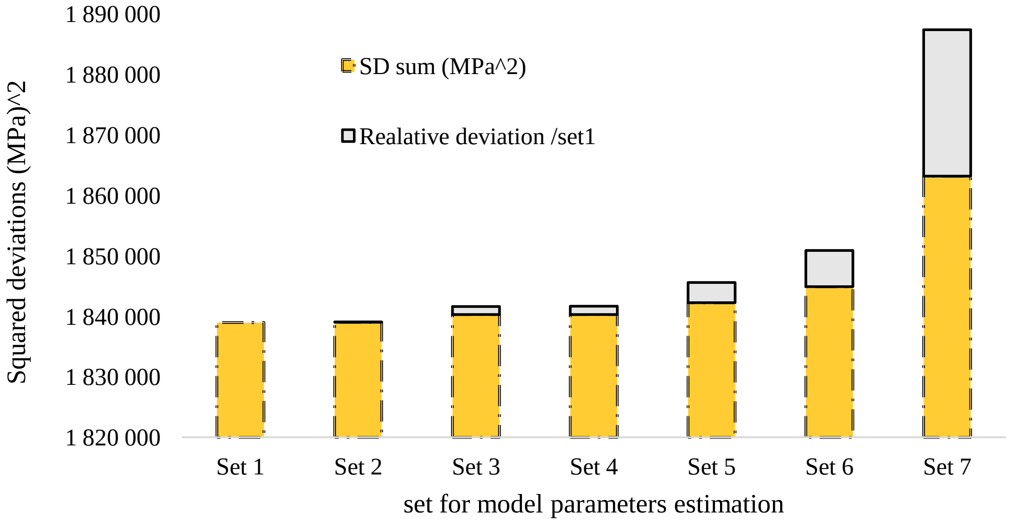

Fig. 2 Comparison of obtained model parameters accuracy to reduce Young's modulus list length

Table 5. Model parameters for sets 1 to 7 used to reduce the list length of plunger penetrations

| **Set** | **K1** | **K2** | **K3** | **Correlation coefficient R** |
|---|---|---|---|---|
| Set 5 | 1,648 | 0,986 | 1,001 | 0,997 |
| Set 8 | 1,648 | 0,987 | 1,001 | 0,997 |
| Set 9 | 1,647 | 0,987 | 1,001 | 0,997 |
| Set 10 | 1,647 | 0,987 | 1,000 | 0,997 |
| Set 11 | 1,647 | 0,987 | 1,000 | 0,997 |

| | | | | |
|---|---|---|---|---|
| Set 12 | 1,647 | 0,987 | 1,000 | 0,997 |

In this set, plunger penetration takes 5 values, starting from 0,6 mm to 3 mm by a step of 0,6 mm, Young's modulus takes 5 values too, starting from 200 MPa to 1000 MPa by a step of 200 MPa. In addition, Poisson's ratio takes four values from 0,15 to 0,45 by a 0,1 step. To reduce the number of values considered here, we remove the first value, and we see the accuracy of the obtained solution of problem (5). The solution is: $k_1 = 2{,}062$, $k_2 = 0{,}698$ and $k_3 = 1{,}000$ with $R = 0{,}999$. When these values are used and compared with set 1 data, the SDS is 3 times more than the minimal SDS of set 1. Removing the second value (i.e. 0,25), the SDS obtained is equal to 27 times the minimal SDS. These tests show that the reduction of Poisson's ratio list length affects the accuracy of the model, and, according to these results, set 11 offers a compromise between accuracy of the model and time consumption, also it is chosen as the representative set for RL-CBR analysis. The model (4) is written as (7).

To compare the accuracy of this solution with previous studies, Table 6 presents parameters' values of previous studies and the ratio of obtained SDS per the reference SDS obtained for set1. After this comparison, the use of the solution of Araya [2] induces a model two times less accurate than the solution of the present study. Other solutions are less accurate with respect of set1 data. It is noted that for this study, a particular model for specimen mould contact is considered: hard for normal contact and frictionless for the tangential one. Other studies [2, 11, 13] adopted other contact models available in ABAQUS software.

$$E_{eq} = \frac{1{,}647(1 - \nu^{0{,}987})\sigma_p \frac{d}{2}}{u} \qquad (7)$$

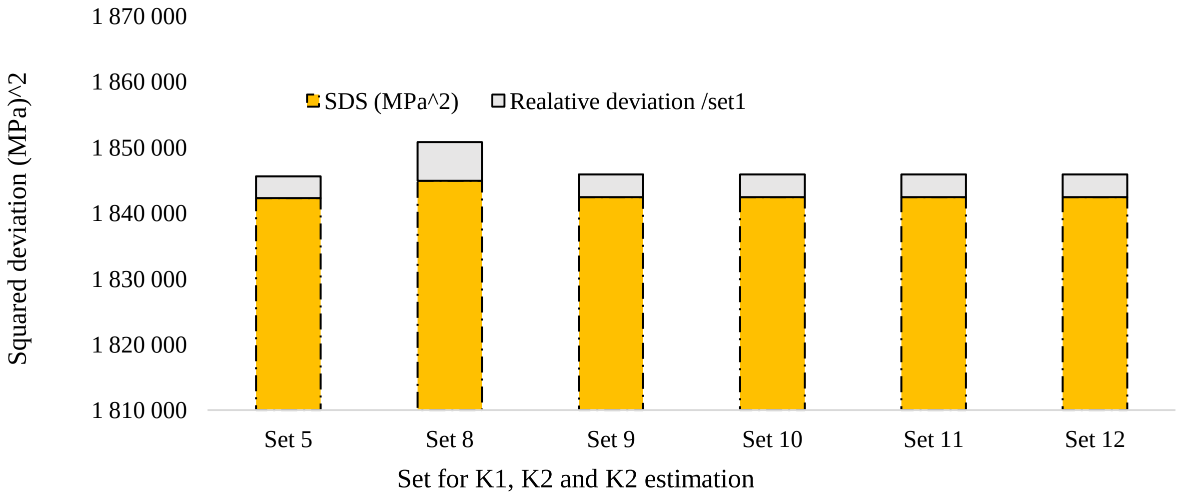

Fig. 3 Comparison of obtained model parameters accuracy to reduce plunger penetrations list length

Table 6. Comparison with previous studies

| **Study** | **Friction** | **K$_1$** | **K$_2$** | **K$_3$** | $^{SDS}/_{SDSset\ 1}$ |
|---|---|---|---|---|---|
| Present study (set 11) | No- friction | 1,647 | 0,987 | 1,000 | 1 |
| [13] | No- friction | 1,377 | 1,552 | 1,056 | 6 |
| [2] | Intermediate friction | 1,513 | 1,104 | 1,012 | 2 |

| | | | | | |
|---|---|---|---|---|---|
| [11] | No- friction | 1,797 | 0,889 | 1,098 | 8 |
| [11] | Full-friction | 1,375 | 1,286 | 1,086 | 10 |

# 5. Conclusions and outlooks

It seems to be clear that the adopted model offers an accurate estimation of equivalent modulus based on RL-CBR test. This study proves that reducing the size of the initial sample of materials and plunger's penetrations can gives an accurate estimation of the equivalent modulus model's parameters. The number of necessary analysis is reduced from 4800 to 100 without decreasing much the accuracy of the model. This reduction is made by reducing the length of Young's modulus list and that of plunger penetrations list. When we have tried to reduce Poisson's ratio list length, the accuracy of model has significantly dropped, so we kept the initial list. This study will continue through an experimental validation of the derived equation; on the first hand, using RL-CBR on standard samples with known mechanical characteristics, on the second hand, experimental results of RL-CBR will be compared to that of repeated load triaxial tests for many UGM after the development of transfer functions for RL-CBR with strain gauges. Equivalent modulus derived from RL-CBR will be compared with resilient modulus of RLT test ant the plate in-situ test, then, equivalent modulus can be used for the design of flexible pavement in the structural design road pavement

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