Coupled effects of solute transport, osmotic pressure and hydrodynamic instabilities in rotating dynamic filtration : Taylor-Couette as model set-up

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Résumé :

Ce travail porte sur l'analyse de stabilité linéaire et la simulation numérique directe d'un écoulement de type Taylor-Couette transportant un scalaire entre deux cylindres semi-perméables. Cette étude est motivée par l'utilisation des instabilités hydrodynamiques (les vortex de Taylor) dans les procédés de filtration afin de limiter les phénomènes d'accumulation de la matière retenue au niveau de la membrane. Le couplage entre les instabilités hydrodynamiques, le transport d'un soluté et la pression osmotique due à la formation d'une couche limite de concentration à l'une des parois, est étudié en présence d'un écoulement radial transmembranaire. Une étude paramétrique est utilisée pour déterminer les effets combinés de la diffusion moléculaire, de l'advection de la couche limite de concentration par l'écoulement transmembranaire, de la pression osmotique et du rapport des rayons sur la dynamique des instabilités centrifuges. L'analyse de stabilité montre que ce mécanisme et le rapport des rayons des deux cylindres modifient fortement les conditions critiques d'apparition des instabilités. Une validation des résultats analytiques est faite en utilisant un code numérique basé sur les méthodes spectrales.

Abstract :

In this work, linear stability analysis and Direct Numerical Simulations (DNS) are used to investigate the dynamics of Taylor-Couette flow carrying a solute between two permeable cylinders. This study is motivated by making use of the hydrodynamic properties (Taylor vortices) in filtration processes in order to prevent the accumulation of retained materials on the membrane. The coupling between hydrodynamic instabilities, membrane transfer and osmotic pressure related to the concentration boundary layer forming near the membrane is examined in the presence of a radial in- or outflow through membranes. A parametric study is used to characterize the combined effect of molecular diffusion, advection of the concentration boundary layer and osmotic pressure on the flow dynamics. Linear stability analysis shows that this mechanism and the radius ratio alter the critical conditions above which Taylor vortices appear. These analytical results are compared to recent DNS based on a dedicated code using spectral methods that shows a good agreement with critical conditions.

Mots clefs : Taylor-Couette, cylindres perméables, transport du soluté, pression osmotique, couche limite de concentration

1 Introduction

Filtration processes separating a suspension or a solute from a carrying fluid (solvent) are ubiquitous in engineering applications but their performances are known to be reduced by the accumulation of retained materials at the surface of the semi-permeable membrane. As an example of these accumulation processes, a concentration boundary layer forms and leads to the appearence of an osmotic pressure that tends to cancel out the operating pressure driving the solvent across the membrane. To ensure an efficient mixing of the solution and abate this concentration boundary layer forming close to membrane, it is interesting to rely on hydrodynamic instabilities. The Taylor-Couette set-up provides an ideal test case to optimize the mixing, as the flow stability has been thoroughly studied in such configuration. This Taylor-Couette flow consists of a viscous fluid confined between a rotating inner cylinder and a fixed outer one. Taylor [1] demonstrated that above a critical rotation rate of the inner cylinder, centrifugal instabilities emerge in the form of toroidal vortices in the gap. Since Taylor's study, several investigations have focused on the structure and dynamics of the vortices ([2, 3] for instance among many others). A radial through-flow imposed between two rotating porous cylinders filled with pure solvent (without solute), is already known to affect the stability of Taylor-Couette flow, as shown [4]. A radial inward flow or strong radial outward flow stabilizes the flow while a weak radial outward flow has a destabilizing effect on the flow [5, 6]. In our specific application with a solute and semi-permeable membranes, however, no quantitative studies exist to examine the coupling between mixing, osmotic pressure, and instabilities. This work aims at understanding the coupling between the concentration boundary layer forming near the membrane via the osmotic pressure, in the presence of a radial through-flow, and the dynamics of the centrifugal instabilities in a Taylor-Couette set-up. We probe the case when a solute is present in a Taylor-Couette cell in its simplest form where both cylinders are semi-permeable as sketched in figure 1.(a). A radial through-flow is imposed and, depending on the radial flow direction, pure solvent permeates through the membrane of the inner or the outer cylinder while the solute is retained at the surface.

2 Linear theory

2.1 Base state

In the situation considered here, a Newtonian fluid transporting the solute is maintained between two porous concentric cylinders of inner and outer radii R_1 and R_2. The inner cylinder is rotating at angular speed Ω while the outer one is fixed. An imposed radial in- or outflow drives the solvent through both cylinders and the solute is totally rejected by the membrane. The velocity field, pressure field and concentration field satisfy the continuity, incompressible Navier-Stokes and scalar transport equations. The non-dimensionalization of the problem leads to introduce non-dimensional parameters : the radial Reynolds number $\alpha = uR_1/\nu$ where ν is the kinematic viscosity, the radius ratio $\eta = R_1/R_2$ and the Taylor number $Ta = R_1\Omega d/\nu$ where d is the gap width $d = R_2 - R_1$. Moreover, specific boundary conditions are applied on both cylinders to account for the coupling of the radial transmembrane velocity U, the pressure P and the concentration C (1a), (1b) and the complete rejection of the solute at membranes(1c),(1d). Note that the pressure difference between P_{in} and P_{out} is the working pressure accross permeable cylinders. These conditions introducing extra non-dimensional parameters : the non-dimensional permeability of both membranes σ, the chemical activity scaling the magnitude of the

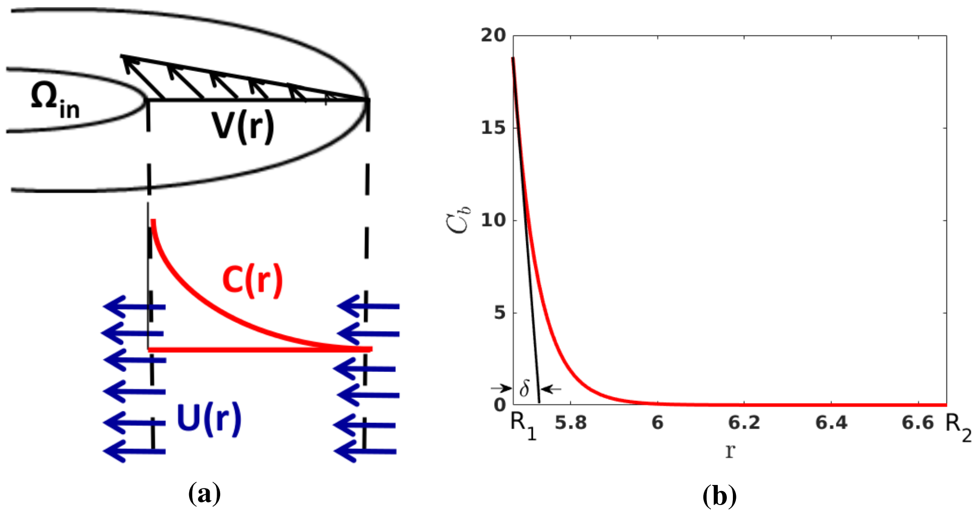

FIGURE 1 – (a) : Sketch of the configuration considered here : radial, azimuthal components of the velocity field U, V and concentration profile C showing concentration boundary layer formed by a radial inflow through two porous cylinders, as a function of radial direction r. (b) : the base concentration field with the thickness of the concentration boundary layer δ in the gap for $\eta = 0.85$, $\alpha = -0.1$ and $Sc = 1000$.

osmotic pressure $\chi = RTd^2C_0/M\rho\nu^2$ (R : the universal ideal gas constant, T : the solution temperature, C_0 : the molar concentration reference value, M : the molar mass of the solute, ρ : the fluid density), and the Schmidt number $Sc = \nu/D$ where D is the solute molecular diffusion.

$$U\left(R_1\right) = \sigma(P_{in} - P\left(R_1\right) + \chi C\left(R_1\right)) \tag{1a}$$

$$U\left(R_2\right) = -\sigma(P_{out} - P\left(R_2\right) + \chi C\left(R_2\right)) \tag{1b}$$

$$U\left(R_1\right) C\left(R_1\right) - \frac{1}{\mathrm{Sc}} \frac{\partial C\left(R_1\right)}{\partial r} = 0 \tag{1c}$$

$$U\left(R_2\right) C\left(R_2\right) - \frac{1}{\mathrm{Sc}} \frac{\partial C\left(R_2\right)}{\partial r} = 0 \tag{1d}$$

Besides, no slip conditions for the velocity are applied on both cylinders. Seeking for a time-independent, axially and azimuthally invariant solution of the non-dimensional equations, we obtain the base state. An example of the base concentration field in the presence of a radial inflow ($\alpha = -0.1$) is shown in figure 1.(b), for $\eta = 0.85$ and $Sc = 1000$. Depending on the radial flow direction (inflow or outflow), a concentration boundary layer builds up near the inner or the outer cylinder. The thickness of this concentration boundary layer δ (figure 1.b), is inversely proportional to the combined effect of advection and molecular diffusion which is described by the Peclet number $Pe = \alpha Sc$.

2.2 Critical conditions

The linear stability analysis of the previous base state predicts the critical conditions of the centrifugal instabilities in the form of the critical Taylor number Ta_{crit} above which toroidal vortices with the axial wavenumber k_{crit} become unstable. Figure 2 depicts these critical conditions as a function of Schmidt number Sc and chemical activity χ (molecular diffusion and osmotic pressure, respectively) in the case of a narrow gap ($\eta = 0.85$) and an imposed radial inflow ($\alpha = -0.1$, the concentration boundary layer

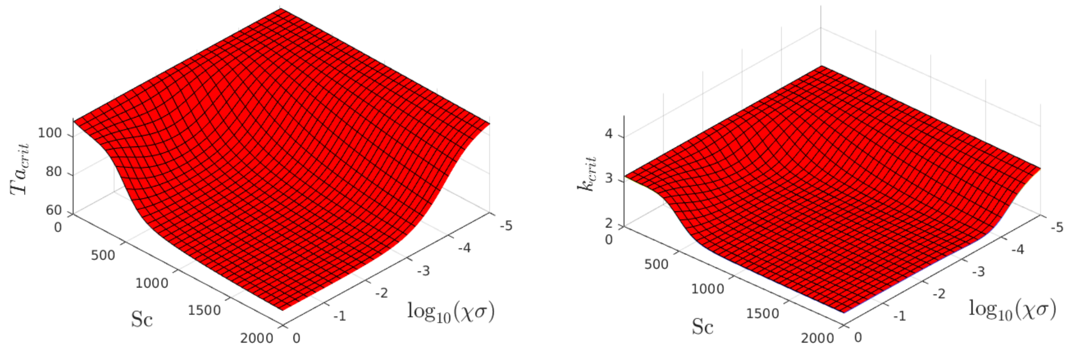

FIGURE 2 – Analytically obtained critical conditions : critical Taylor number Ta_{crit} and critical axial wavenumber k_{crit} as a function of Schmidt number Sc and chemical activity-permeability product $\chi\sigma$ in logarithmic scale for $\eta = 0.85$ and $\alpha = -0.1$.

builds up at the inner cylinder). For $Sc = 2000$ and $\chi\sigma = 1$, the critical Taylor number is substantially decreased to $Ta_{crit} = 67.61$ compared to the case without solute $Ta_{crit} = 108.44$ (for $Sc = 0$). Besides, increasing the Schmidt number or the osmotic pressure also increases the characteristic size of the vortices along the axial direction, in relation with a decreasing k_{crit}. With a fixed value for permeability σ, the decrease of the instability threshold is already observed for small values of the chemical activity χ. When a limit value of the chemical activity χ and the Schmidt number Sc is reached, critical conditions are no longer depending on osmotic pressure and molecular diffusion.

3 Direct Numerical Simulations

To validate analytical results, a numerical code based on spectral methods has been used to model the situation considered here. This code has already been implemented for flows without solute between two permeable walls [7] and it is extended in this study to solve the scalar transport equation and include

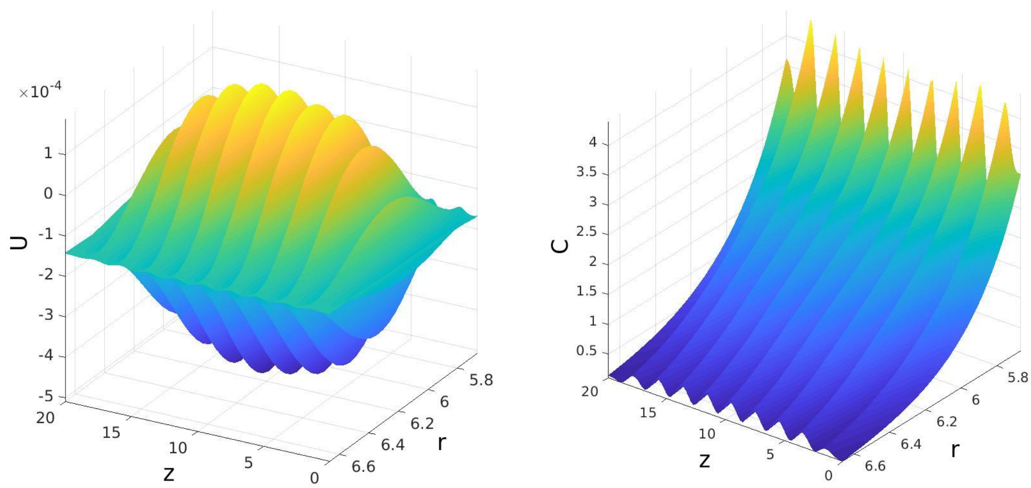

FIGURE 3 – DNS : Radial velocity U and concentration field C in the gap for $\eta = 0.85$, $\alpha = -0.1$, $Sc = 200$, $\chi\sigma = 10^{-3}$ and $Ta = 106$.

the radial transmembrane velocity-osmotic pressure coupling and the total rejection of the solute as both expressed as boundary conditions. Analytical results and numerical simulations are compared in the case of narrow gap $\eta = 0.85$, radial inflow $\alpha = -0.1$, $Sc = 200$ and $\chi\sigma = 10^{-3}$. For this set of parameters values, analytical value for critical Taylor number is $Ta_{crit} = 105.84$. The numerical simulations in a single domain of length $L = 20$ at Taylor number $Ta = 106$ as shown in figure 3 (the numerically computed radial component of the velocity field U and the concentration field C) indicate the presence of toroidal vortices. At $Ta = 105$, the DNS shows that the perturbation is damped so the flow remains laminar. For other set of parameters, further numerical simulations are compared to analytical predictions that shows a good agreement with critical conditions.

4 Conclusions and outlook

Using analytical approach, the osmotic pressure induced by the build up of a concentration boundary layer has been found to substantially impact the stability of a Taylor-Couette flow bounded by permeable cylinders. In addition, this result has been validated by a numerical code including more realistic boundary conditions (1). The combined effect between mass transfer and osmotic pressure promotes centrifugal instabilities by decreasing critical conditions. In this study, further analytical and numerical simulations are in progress to probe the impact of radius ratio on flow stability and to describe the non-linear behaviour of instabilities in the presence of solute and osmotic pressure.

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