On stability of the regular vortex polygon in two-layer fluid and two-fluid plasmas

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Abstract :

The models of point vortices motion in two-layer rotating fluid and two-fluid plasmas are considered. These two different physical models are described by the same system of equations for different values of a parameter β :

$$\dot{\bar{z}}_k = 2iH_{z_k}, \quad \dot{z}_k = -2iH_{\bar{z}_k}, \qquad k = 1, \ldots, N, \tag{1}$$

$$H = -\frac{\Gamma}{4\pi} \sum_{1 \le j < k \le N} W\left(|z_j - z_k|\right), \qquad W(\xi) = \ln \xi - \beta \mathrm{K}_0(\xi). \tag{2}$$

Positive values of the parameter β correspond to the geostrophic model of a two-layer fluid [1, 2]. All vortices lie in the same layer (to be definite, in the lower one). The thickness of the upper layer is h_1 and the lower layer has thickness h_2. The condition $h_1 + h_2 = 1$ is valid. The parameter $\beta = h_1/h_2$ is the ratio of the layers thicknesses h_1 and h_2.

Negative values of the parameter β correspond to Alfven model of two-fluid plasma [3].

The Hamiltonian equations (1), (2) describe the motion of the system of N point of vortices with equal intensity Γ.

The system (1), (2) has exact solution

$$z_k = e^{i\omega_N t} u_k, \quad u_k = Re^{2\pi i(k-1)/N}, \quad R > 0, \quad k = 1, \ldots, N, \tag{3}$$

corresponding to the stationary rotation of the configuration of N point vortices lying at the vertices of the regular N-gon. Here R is the radius of the circle on which the vortices are located, ω_N is the angular velocity.

The stability of the stationary rotation (3) is studied. The stability of this solution is interpreted as orbital stability. The instability is instability of system reduced equilibrium. The quadratic part of the Hamiltonian and eigenvalues of the linearization matrix are studied analytically.

The parameters space (N, R, β) is divided on three parts : (**A**) the domain of stability in an exact nonlinear setting, (**B**) the linear stability domain, where the stability problem requires the nonlinear analysis, and the instability domain (**C**).

If $\beta > 0$, in the case of the two-layer quasigeostrophic model the stability is studied for any $N \geq 2$. The results are published in [4]. The case (**A**) takes place for $N = 2, 3, 4$ for all possible values of parameters R and β. In the case of $N = 5$ we have two domains : (**A**) and (**B**). In the case $N = 6$ part (**B**) is curve, which divides the space of parameters (R, β) into the domains : (**A**) and (**C**). In the case of $N = 7$ there are all three domains : (**A**), (**B**), and (**C**). The instability domain (**C**) takes place always if $N = 2n \geqslant 8$. In the case of $N = 2\ell + 1 \geqslant 9$ there are two domains : (**B**) and (**C**). The results of theoretical analysis are confirmed by numerical calculations of the vortex trajectories.

If $\beta < 0$, the stability problem for two-fluid plasmas model are considered for $N = 2, \ldots, 9$. The stability diagram has a more complex form. We have two domains (**A**) and (**B**) in the cases of $N = 2, 3$, and there are all three domains (**A**), (**B**), and (**C**), if $N = 4, \ldots, 9$. The results obtained are consistent with the numerical calculation of paper [3]. Part of the results included in the paper [5].

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Mots clefs : N-vortex problem ; Point vortices ; Two-layer fluid ; Stability ; Two-fluid plasma

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