Key Characteristics Identification based on Tolerance Allocation by Iso-Sensitivity

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Abstract

In the design phase of manufactured products, it is necessary to specify the dimensions with critical importance, which are referred to as the Key Characteristics (KCs). A mechanical system involves multiple functional requirements. In this study each functional requirement considered includes a chain of dimensions, which are modelled as random variables. In industrial practices the use of the Failure Mode Effects and Criticality Analysis (FMECA) allows the identification of the critical functional requirement, and then all the involved parts are set as KCs. KCs cause high production costs since they need special attention, so it is required to reduce their number and specify only the necessary ones. There is no common method for the identification of the KCs in a critical functional requirement. The strategies followed for this purpose differs from a company to another. The strategy to be followed for the identification of KCs should take into consideration the tolerance intervals, the capability requirements and the mathematical formulation of the topological loop comprising the studied dimensions. The method proposed in this article consists of considering that all the dimensions involved in a critical functional requirement have equal sensitivities, and then estimate the corresponding tolerance intervals to match what is called "Iso-sensitivity tolerance allocation". This is done under the conditions that are required by the companies for a high quality production. In this study the condition to be satisfied is that the Non Conformity Rate (NCR) should be equal to a target value. Therefore, this is an optimization problem with an objective to have equal sensitivities and equality constraints on the NCR to estimate the tolerance intervals. First a design is proposed and then an optimal design is found after applying the proper optimization algorithm. The estimated tolerance intervals are then compared to reference tolerance values that can be achieved by the manufacturer. So when a tolerance interval for a corresponding dimension cannot be achieved, it is considered as a KC.

Keywords : Key characteristics, Tolerance allocation, Iso-sensitivity

1 Introduction

Geometrical imperfections are present in any manufacturing process. The dimensions of manufactured parts differs from the ones specified by the designers. Such imperfections have unfavourable effects on the performance of mechanical systems. The designers usually associate tolerance intervals and capability requirements with each dimension in a component. The designers are also concerned to specify the most critical dimensions or what is called the Key Characteristics (KCs). Key Characteristics are defined as the parts which variation significantly affects the performance of the final product [9]. A mechanical system has multiple functional requirements. If the functional requirement is not respected, the performance of this system is affected. The industrial practice consists of using the Failure Mode Effect and Criticality Analysis (FMECA) [5]. FMECA is a method to identify the critical functional requirements that have an important effect on the system. Once the functional requirements are set, all the corresponding dimensions are considered as Key Characteristics. However, not all the dimensions have the same impact on the system. Therefore the current practice is very conservative. A Key Characteristic needs special attention, consequently a high number of KCs engenders high costs of production. There is need to reduce the number of KCs to prevent the problem of the high costs. Thus, the dimensions with no influence should be excluded. This study proposes a method that allows the identification of the KCs based on the tolerance allocation by Iso-sensitivity.

Section 2 and Section 3 present briefly the formulation of the Non-conformity rate and the sensitivity methods that will be the tools to use in the proposed approach presented in Section 4. Section 5 is a numerical application of the proposed method and finally Section 6 presents the conclusions and perspectives.

2 Formulation of the Non Conformity Rate

The quality requirements of a mass produced systems are needed to assure that the assembled product is robust with respect to manufacturing variability. A defect probability is allowed for each functional requirement and is expressed in parts per million (ppm). The functional requirement is expressed in terms of the geometric dimensions such as:

$$Y = f(\mathbf{X}) \tag{1}$$

where $\mathbf{X}$ is the vector of the dimensions involved in the functional requirement. The functional requirement should be bounded by the LSL_Y and USL_Y which are respectively the lowest and the upper specification limits. In this article the defect probability is referred as the Non Conformity Rate (NCR). It is estimated by determining the probability that Y, the functional requirement of the assembly, will be not between the required bounds such as:

$$NCR = P(Y \notin [LSL_Y; USL_Y]) \tag{2}$$

A part dimension X_i is defined by a target value T_i and a corresponding tolerance interval t_i . It is assumed to follow a Gaussian distribution A production batch corresponding to X_i is defined by a standard deviation σ_i and a mean value μ_i measured on the X_i batch sample. The mean shift δ_i is deduced by $\delta_i = \mu_i - T_i$.

The process is considered under statistical control, therefore the behaviour of the manufacturing process is controlled by two capability indices, C_p and C_{pk} that are defined in Equations (3,4) [8].

$$C_{pi} = \frac{t_i}{6\sigma_i} \tag{3}$$

$$C_{pki} = \frac{t_i/2 - |\delta_i|}{3\sigma_i} \tag{4}$$

The given formulas of the capability indices are only used when the variables follow a normal distribution. More general definitions of C_p and C_{pk} are provided in [1] applied for different types of distributions. The production batch must satisfy the capability requirements, that is $C_{pi} \geq C_{pi}^{(r)}$ and $C_{pki} \geq C_{pki}^{(r)}$. These requirements come from the industrial requirements of the manufacturers and the customers.

3   Overview on the sensitivity analysis methods

The term sensitivity analysis is defined differently in various technical fields. To give an accurate definition of the sensitivity analysis, the output of the model need to be specified. The sensitivity analysis is defined in [6] as "*the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input*". Many techniques are used for sensitivity analysis (SA), they are classified in different categories depending on the problem setting: local sensitivity methods, and global sensitivity methods. [2] is a detailed review on all sensitivity analysis methods. [4] is also a review on global sensitivity analysis methods. Local sensitivity considers the influence of the input parameters in the vicinity of a reference point. Mathematically, the local sensitivity analysis consists in estimating the partial derivative with respect to each variable input that characterizes the effect on the random value $y(\theta)$ of a perturbation on input near a nominal value $\theta^{(0)}$. The sensitivity index is expressed by the following relation:

$$S_i(\boldsymbol{\theta^{(0)}}) = \frac{\partial y}{\partial \theta_i}(\boldsymbol{\theta^{(0)}}) \tag{5}$$

Global sensitivity analysis focuses on the output uncertainty over the entire range of values of the input parameters. It explores the parameter space so that robust sensitivity measures are provided in the case of non-linearity and in the presence of interactions between the parameters. Global sensitivity methods give more detailed results than local ones. An example of the global sensitivity methods is the Sobol' method which is one of the widely used global sensitivity analysis methods based on variance decomposition [7]. It is used in the context of tolerance analysis in some works such as [10, 11].

4   Formulation of the tolerance allocation by Iso-sensitivity problem

The proposed method is a step forward to be considered after the identification of the critical functional requirements. As mentioned in the introduction, the identification of KCs is done by specifying the critical functional requirements using FMECA and then considering all the involved dimensions as KCS. Based on this idea, the proposed method suggests to set the sensitivities of all the dimensions in the critical functional requirement as equal. Then a target NCR to be respected during the manufacturing

is imposed. Finally, optimal tolerance intervals associated to each dimension are to be estimated. This approach can be formulated as an optimization problem that can be expressed as:

$$\boldsymbol{t} = Argmin \sum_i^n (S_i(t) - \overline{S}(t))^2 \qquad (6)$$

$$\text{s.t.} \quad NCR(t) - NCR_{target} = 0 \qquad (7)$$

where $\boldsymbol{t}$ is the vector involving all the tolerance intervals associated to the dimensions, n is the total number of variables, $S_i(t)$ is the sensitivity index associated to each dimension and is a function of the tolerance interval t, and $\overline{S}(t)$ is the mean value of the sensitivities. The sensitivity indices are estimated in this work by the local sensitivity analysis since they are easy to implement. They are the sensitivity of the $NCR(t)$ to the tolerance intervals.

The estimated tolerance intervals are then compared to reference values that are provided by the company. The reference values are the minimum tolerances that the machine can attain. That is, the machine cannot manufacture dimensions with tolerances below the reference values. Therefore, the dimensions having tolerances that are tighter than the reference ones are critical since they cannot be produced properly in such a way to have equal sensitivities. Consequently, these dimensions are critical so they are set as KCs. When the tolerance intervals are changed, the optimization problem can be repeated with the new values until the optimal intervals are reached.

5 Numerical Applications

The proposed method is applied on two different examples, a linear stack-up example and a non linear clutch mechanism.

5.1 Linear stack-up example

A linear stack-up composed of five parts is considered as shown in Figure.1. The functional requirement can be expressed as:

$$J_a = a_5 - (a_1 + a_2 + a_3 + a_4 + a_5) \in [0, 0.8] \qquad (8)$$

The objective of this problem is to estimate the tolerance intervals corresponding to each dimension with the condition that corresponding NCR should be equal to a target value ($NCR_{targrt} = 1.10^{-5}$). The sensitivity indices are calculated using the local sensitivity methods. The input data of the problem are given in Table. 1. The corresponding tolerance intervals are given in Table. 2. The calculated tolerance intervals should be compared to reference values provided by the company. The dimensions having an unreachable tolerances are critical, therefore they are considered as a KCs.

Table 1: Input data for the linear stack-up example

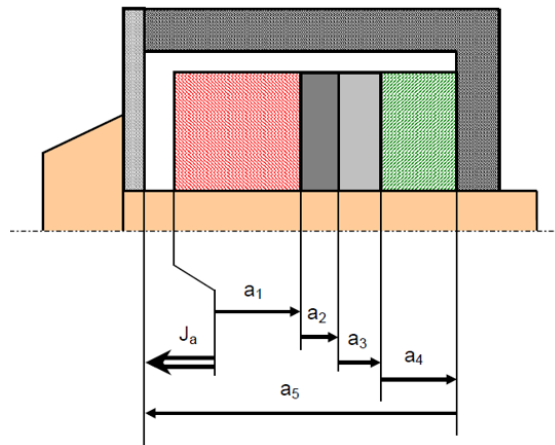| Dimensions	Mean Values	$C_p^{(r)} = C_{pk}^{(r)}$
a_1	20	1.33
a_2	4	1.33
a_3	5	1.33
a_4	10	1.33
a_5	39.4	1

Figure 1: Representation of the linear stack-up example

Table 2: Corresponding tolerance intervals for the stack-up example

Dimensions	Tolerance Intervals by Iso-sensitivity
a_1	0.316
a_2	0.316
a_3	0.316
a_4	0.316
a_5	0.280

5.2　Non linear example: Clutch mechanism

Another example is the non linear clutch mechanism [3]. The angle ϕ is expected to vary due to manufacturing variations in the clutch component dimensions. The corresponding functional requirement is:

$$\phi = cos^{-1}\left(\frac{A+C}{E-C}\right) \in [6.4, 7.6] \tag{9}$$

To simplify the calculation, this function is linearised by Taylor series. The target value of the NCR is 1.10^{-4}. The input data of the problem are given in Table.3 and the results of the optimization are presented in Table. 4. As mentioned in the first example, the calculated tolerance intervals are to be compared to reference values provided by the company. The dimensions having an unreachable tolerances are critical, therefore they are considered as a KCs.
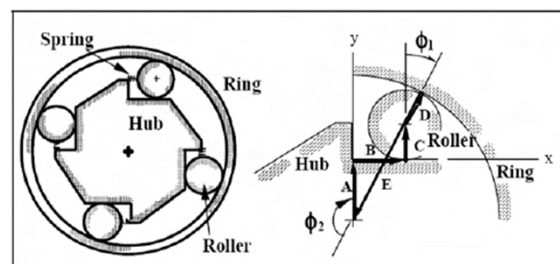

Figure 2: Representation of the non linear clutch mechanism [3]

Table 3: Input data for the clutch mechanism

Dimensions	Mean Values	$C_p^{(r)} = C_{pk}^{(r)}$
A	27.645	1
B	11.43	1
C	50.8	1

Table 4: Tolerance Intervals for the clutch mechanism

Dimensions	Tolerance Intervals
A	0.306
B	0.08
C	0.302

6 Conclusions and perspectives

This paper presents a new approach to reduce the number of KCs in the critical functional requirements. The method is based on the concept of tolerance allocation by Iso-sensitivity. It allows the estimation of the tolerance intervals by considering equal sensitivities for the dimensions and under the condition that the NCR should have a target value. It is then formulated as an optimization problem that estimates the tolerance intervals. The proposed approach is applied on two numerical examples, a linear stack-up and a non-linear clutch mechanism. Tolerance intervals are calculated in both cases. The obtained tolerances should be compared to reference values already specified by the company. When the tolerances cannot be attained by the manufacturer, the corresponding dimensions are considered as KCs. The sensitivities in this work are calculated using the local methods. In later steps, the sensitivity indices will be calculated by global sensitivity analysis.

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