Resolution of a forming problem with the Asymptotic Numerical Method in Arbitrary Lagrangian Eulerian Formulation

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Résumé :

Cette communication concerne l'étude de la résolution de l'équation d'advection avec la Méthode Asymptotique Numérique. Cette équation doit être résolue lorsqu'une formulation Arbitraire Lagrangien Eulérien est utilisée pour résoudre un problème incluant de la plasticité par exemple.

Abstract :

This communication is devoted to the resolution of the advection equation with the Asymptotic Numerical Method. This equation has to be solved when an Arbitrary Lagrangian Eulerian formulation is used to solve a forming problem including for instance plasticity.

Mots clefs : Méthode Asymptotique Numérique, ALE, advection

1 Introduction

In the manufacturing processes of metallic sheets, such as levelling, the sheet is deformed by a succession of rollers. This kind of process implies contact with friction and elasto-plasticity. To have accurate results in the context of the finite element method, it is necessary to have a fine mesh in the contact zones. As the metallic sheets are conveyed along the production line, each material point of the surface enters into contact with rollers during the process. If classical lagrangian formulations are used, it implies that the mesh needs to be fine all along the sheet. Consequently, the computational time becomes very important. This problem can be handled by using an ALE formulation : the mesh is nor fixed to the material domain neither fixed in space like in Eulerian formulations. A lot of examples are available in the literature concerning the ALE formulation for such problems.

Another challenge is to solve the non-linear equations (contact, plasticity). We propose here to handle the involved nonlinearities with a high order continuation procedure based on the Asymptotic Numerical Method (ANM) [1]. At each step of the continuation procedure, the variables of the problem are represented by truncated power series whose terms are solutions of a series of well-posed linear problems having the same tangent matrix. A software called Manitoo has been developed within this framework and is described in [2].

ALE formulations imply that time derivatives of state variables can not be computed like in Lagrangian computations and a convective term has to be added to the time derivatives as it is classically done in Eulerian computations in fluid mechanics. For instance, the time derivative of the deformation in a given direction $\dot{\varepsilon}$ becomes :

$$\frac{D\varepsilon}{Dt} = \frac{\partial \varepsilon}{\partial t} + \vec{V} \cdot \vec{grad}\varepsilon \tag{1}$$

where the left-hand side is the particular derivative whereas the left-hanside contains the derivative of the deformation in the ALE domain plus a convective term. The term V is the the difference between the velocity of the particule and the velocity of the ALE mesh. Equation 1 is known as the "advection equation". Many studies have been carried out on this topic in the framework of ALE formulation for solid mechanics [3]. Classically two methods are proposed to add this convective effects to the FE codes. The first one is the strong coupling method which consists in solving the real equations including these convective terms. The second one is the weak coupling method : first a lagrangian computation is performed and then a convective phase brings a correction to the results of the lagragian phase.

In the context of ANM, the two methods can be considered. Nevertheless, the methods used to consider the convective terms are very different. With the weak coupling, the various methods developed can be directly used between each branch independently from the ANM context. If we use the strong coupling method, the advection equation has to be included in the constitutive law and solved with the ANM. This is the subject of this paper.

2 Comparison of different schemes for the advection equation in the ANM context

In order to study the accuracy of different schemes to solve the advection equation, let us consider the 1D advection equation :

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \tag{2}$$

In the context of the ANM, a unknown u is developed in a power series

$$u\left(t\right) - u_0 = \sum_{i=1}^{n} u_i t^i \tag{3}$$

where t is taken as the physical time. Consequently, Equation (2) becomes at order p :

$$(p+1)u_{p+1} + c\frac{\partial u_p}{\partial x} = 0 \tag{4}$$

Now, we will consider different finite difference schemes for the spatial derivative to solve this equation. We can notice that no choice has to be done for the time discretization. In what follows, we consider that $x \in [0; 1]$. This domain is discretized with $N_p = 201$ nodes with a constant step Δx. In order to test the performance of the schemes, we will apply this equation to the transport of the continuous function and the rectangular function presented in figures 1 and 2. The considered function gives the values of $u_0^j = u_0(x_j)$. Periodic boundary conditions are applied : $u(0) = u(1)$. The computation is performed with $c = 1$ up to $t = 1$ s so that the function is supposed to have completed exactly one "lap".

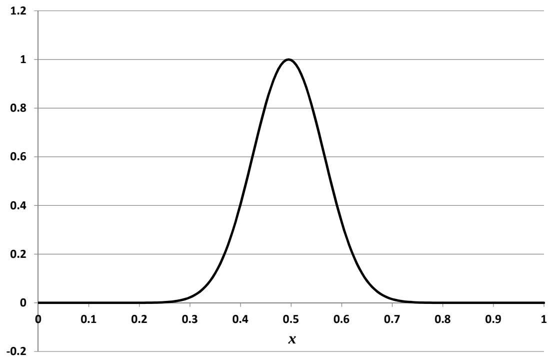

FIGURE 1 – Smooth function

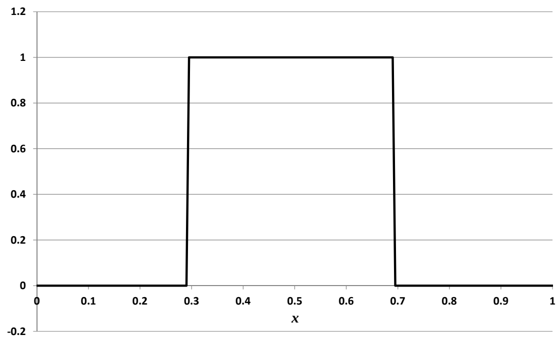

FIGURE 2 – Rectangular function

2.1 Central scheme

First, let us consider the central derivative

$$\left(\frac{\partial u_p}{\partial x}\right)_p^j = \frac{u_p^{j+1} - u_p^{j-1}}{2\Delta x} \tag{5}$$

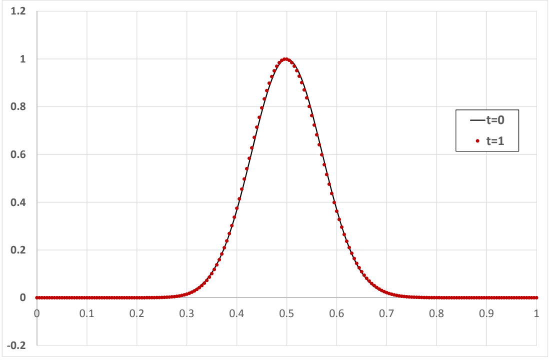

FIGURE 3 – Central : Smooth function (90 steps)

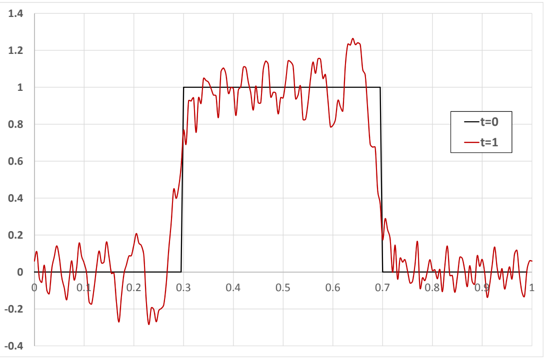

FIGURE 4 – Central : Rectangular function (134 steps)

We find the classical behaviour obtained in usual finite difference computations : the smooth functions are well transported (Fig 3) whereas the transport of discontinuous functions exhibits oscillations (Fig 4).

2.2 Upwind scheme

Classically, we try to use an "upwind" scheme to handle this problem. This corresponds to :

$$\left(\frac{\partial u_p}{\partial x}\right)_p^j = \frac{u_p^{j} - u_p^{j-1}}{\Delta x} \tag{6}$$

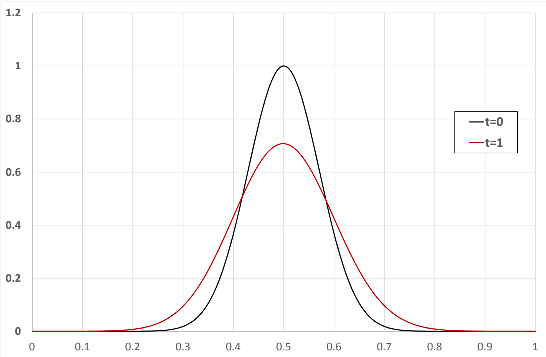

FIGURE 5 – Upwind : Smooth function

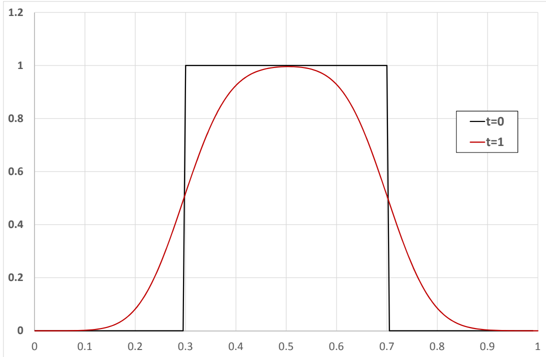

FIGURE 6 – Upwind : Rectangular function

The transport of the rectangular function is better (Fig. 6) but the transport of the smooth function is done with an important diffusion (5). The maximum of the function has decreased by 30%.

2.3 Fromm scheme

To improve the accuraty of the previous scheme, one has to increase the order of the scheme. For classical finite difference approximations, the Lax-Wendroff scheme is often considered. In the ANM context, this scheme can not be used because of the particular treatment of time t in the power series (3). Consequently, we consider the second order scheme of Fromm [4]. From the Taylor expansion of the unknown u, one gets :

$$\left(\frac{\partial u}{\partial x}\right)_p^j = \frac{u_p^j - u_p^{j-1}}{\Delta x} + \frac{\Delta x}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_p^j \tag{7}$$

The Froom scheme consists in taking for the second order derivative the mean between the central second order derivatives in j and $j-1$. This leads to :

$$\left(\frac{\partial u_p}{\partial x}\right)_p^j = \frac{u_p^{j+1} + 3u_p^j - 5u_p^{j-1} + u_p^{j-2}}{4\Delta x} \tag{8}$$

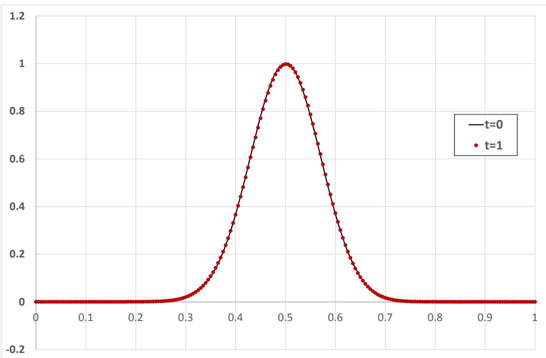

FIGURE 7 – Fromm : Smooth function (46 steps)

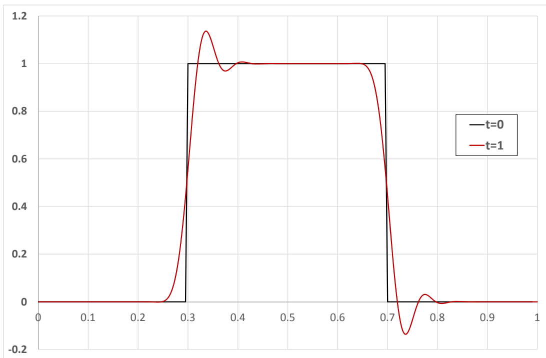

FIGURE 8 – Fromm : Rectangular function (90 steps)

The results are much better, really close for the transport of the smooth function (Fig. 7) and also for the transport of the rectangular function where the are small oscillations near the singularities (Fig. 8).

3 Applications

3.1 Model problem

We consider the following model problem : in two parts of the segment $[0;1]$, we impose a non-zero plastic strain rate $\dot{\varepsilon}^p$ (see Fig. 9). It could correspond to the plasticity induced by 2 rollers on a metallic sheet during leveling. We want to find the plastic deformation $\hat{\varepsilon}^p$ in the ALE configuration considering that all the points are moving at the velocity $c = 1\ m/s$.

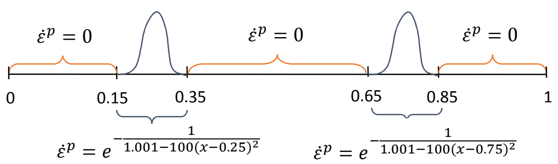

FIGURE 9 – Model problem : prescribed plastic strain rate

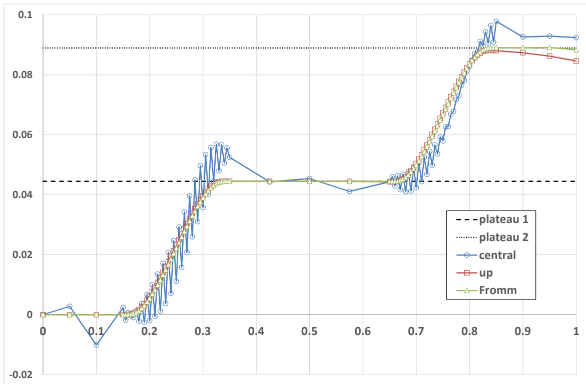

FIGURE 10 – Comparison of $\hat{\varepsilon}^p$ for a non-uniform mesh of 91 points

Considering the convective terms, the following transport equation has to be solved :

$$\dot{\varepsilon}^p = \dot{\hat{\varepsilon}}^p + c\frac{\partial \hat{\varepsilon}^p}{\partial x} \tag{9}$$

In the ANM context, at the order n, it leads to

$$\hat{\varepsilon}^p_{(n+1)} = \varepsilon^p_{(n+1)} - \frac{c}{n+1}\left(\frac{\partial \hat{\varepsilon}^p}{\partial x}\right)_n \tag{10}$$

One of the advantages of ALE for the modelling of levelling is that the mesh can only be refined in the areas in contact with the rollers since the points in contact are nearly the same in the ALE domain during the whole simulation. We now test the different methods to solve the model problem with a non-uniform mesh. The mesh is coarse (0.05) where $\dot{\varepsilon}^p = 0$ whereas it is finer (0.005) where $\dot{\varepsilon}^p \neq 0$. The central scheme shows oscillations with this non-uniform mesh (Fig. 10). The two other methods are quite similar even if the Fromm scheme finds a better approximation of the second plateau.

3.2 Coupling with the incremental plasticity equations

We can couple now the previous work with the incremental plasticity equations developed in the ANM framework [5]. Instead of giving the plastic strain rate, we now impose the strain rate and search for the plastic strain over the domain.

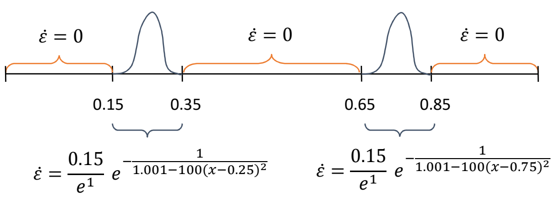

FIGURE 11 – Model problem : prescribed total stain rate

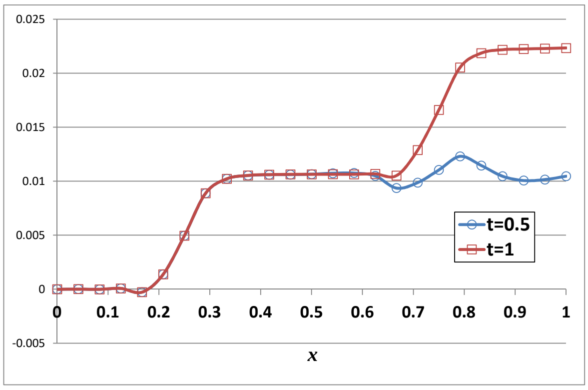

FIGURE 12 – $\hat{\varepsilon}^p$ obtained with the Fromm scheme

4 Coupling with Finite Element

We want to study the 2D example presented in Fig. 13. The 2D solid moves in the x-direction whereas a distributed force is applied at a constant x-position. Incremental plasticity is considered.

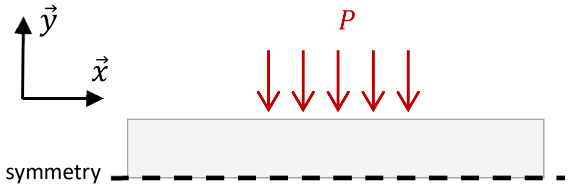

FIGURE 13 – 2D FE problem considered

The convective terms only appear in the constitutive law which is used at the Gauss points of the elements. The mesh used is structured so that we can create lines of Gauss points in the x-direction. Taking a null mesh velocity in x-direction, we have an ALE velocity only in the x-direction and we can apply the schemes presented in the previous sections. The last results will be presented.

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