Global sensitivity analysis of seawater intrusion in fractured coastal aquifers simulated by coupled variable-density flow and discrete fracture network models

B. Koohbor [a], M. Fahs [a], B. Belfort [a], A. Younes [a,b,c], B. Ataie-Ashtiani [d,e], C. T. Simmons [e]

a. Laboratoire d'Hydrologie et Geochemie de Strasbourg, University of Strasbourg/EOST/ENGEES, CNRS, 1 rue Blessig 67084 Strasbourg, France

b. IRD UMR LISAH, F-92761 Montpellier, France.

c. Laboratoire de Modélisation en Hydraulique et Environnement, Ecole Nationale d'Ingénieurs de Tunis, Tunisia

d. Department of Civil Engineering, Sharif University of Technology, PO Box 11155-9313, Tehran, Iran

e. National Centre for Groundwater Research & Training, College of Science & Engineering, Flinders University, GPO Box 2100, Adelaide, South Australia 5001, Australia

koohbor@unistra.fr, fahs@unistra.fr, belfort@unistra.fr, younes@unistra.fr, ataie@sharif.edu, craig.simmons@flinders.edu.au

Abstract

The aim of this study is to use polynomial chaos expansion (PCE) as a tool for performing uncertainty analysis on Seawater Intrusion (SWI) in fractured coastal aquifers simulated using the coupled Discrete Fracture Network (DFN) and Variable-Density Flow models (VDF). Applying VDF-DFN requires a detailed description of fracture parameters, most importantly the geometric characteristics and hydraulic properties of the major fractures. Two fractured configurations of Henry Problem are introduced and investigated throughout the process. The VDF-DFN models are simulated and solved using finite element based framework of COMSOL Multiphysics®. Then, for the mentioned configurations, global sensitivity analysis is performed in 3 steps: (1) experimental design set is generated using quasi Monte-Carlo sampling technic, (2) then, to increase the computational efficiency of the approach, sparse PCEs are built for each model output, and finally (3) Sobol' indices, which are directly derived from the surrogate PCEs coefficients, are used as sensitivity measures to investigate the primary sources of uncertainties in the model outputs. The results can be used as a tool for technical and managerial purposes associated with monitoring, control, and prevention of SWI in fractured coastal aquifers.

Keywords: variable-density flow; discrete fracture network; seawater intrusion; polynomial chaos expansion; global sensitivity analysis

1 Introduction

Seawater Intrusion (SWI) is considered a major threat for the large number of the inhabitants of coastal regions. Adverse effects of this phenomenon can be amplified due to the growing demand for groundwater as a result of population growth and the increasing number of pumping wells. Moreover, the existence of major fractures is the most challenging form of natural heterogeneity that can lead to an intensification of SWI. Fractured Coastal Aquifers (FCAs) are spread in many regions of Europe such as France, Greece, Ireland, and Mediterranean zones. In spite of their prevalence and obviously significant influence on SWI extents, FCAs have been poorly investigated in the literature and are still of high interest for the society of researchers both in Natural Sciences and in Engineering.

The two major approaches for mathematical modelling of SWI are attributed to: (1) Sharp Interface approximation and (2) Variable-Density Flow (VDF) model. VDF is considered a more realistic approach as it delivers a better approximation of the transition zone between freshwater and saltwater by considering the effect of dispersion and diffusion mechanisms on the convective flow. In parallel, among the methods to address flow in fractured porous media, Discrete Fracture Network (DFN) model is the most accurate approach as it does not impose any significant assumption on the physics of the problem. Therefore, combining VDF and DFN can be a realistic and relatively accurate approach to model SWI in FCAs.

The major drawback of applying DFN approach on real-case data is the fact that DFN requires the explicit description of fractures as individual entities with specified geometry and hydraulic properties which is a major challenge for site application. As a result, to analyze the effect of some key features of FCAs on the extents of SWI, the authors of this work suggest performing a global sensitivity analysis based on polynomial chaos expansion (PCE). It allows to identify the major source of uncertainty and then to quantify this uncertainty on the output generated using VDF-DFN model. The results can be of interest for managerial purposes and also for expanding the knowledge on the characteristics of such an approach.

2 Materials and Methods

2.1 Conceptual Model

The conceptual model is based on fractured Henry Problem, suggested by Sebben *et al.* (2015) [1]. The configurations addressed in our work are: (1) Single Horizontal Fracture (SHF) configuration with changing fracture position characterized by d^F, the distance of the fracture from the aquifer top surface (Fig. 1a) and (2) Network of Horizontal Fractures (NOF) configuration with changing fracture spacing with δ^F representing the distance between two consecutive parallel fractures (Fig. 1b). In both configurations, the effect of uncertainty of key hydraulic properties of the fracture on SWI and various relevant metrics are investigated (see Fig. 2). The hydraulic properties are: permeability in the fracture (K^F), fracture aperture (e^F), longitudinal dispersivity in the fracture ($\alpha^F_{L,}$) and longitudinal dispersivity in the matrix ($\alpha^M_{L,}$).

2.2 Governing Equations

Under steady-state condition and based on Boussinesq approximation the VDF model in porous matrix and fracture are as followed:

$$\nabla \cdot \mathbf{q} = 0 \qquad (1)$$

$$\mathbf{q}=-K^{\beta}\left(\nabla h+\frac{\rho-\rho_0}{\rho_0}\nabla z\right) \qquad (2)$$

$$\mathbf{q}\nabla c-\nabla\cdot\left(\varepsilon^{\beta}D_m\mathbf{I}+\mathbf{D}\right)\nabla c=0 \qquad (3)$$

$$\mathbf{D}=\left(\alpha_L^{\beta}-\alpha_T^{\beta}\right)\frac{\mathbf{q}\times\mathbf{q}}{|\mathbf{q}|}+\alpha_T^{\beta}|\mathbf{q}|\mathbf{I} \qquad (4)$$

$$\rho=\rho_0+\Delta\rho.c \qquad (5)$$

The superscript β corresponds to the type of discussed media; it can be replaced by “M” when specifying the equations governing the flow and transport in matrix and also it can be replaced by “F” when dealing with the fracture.

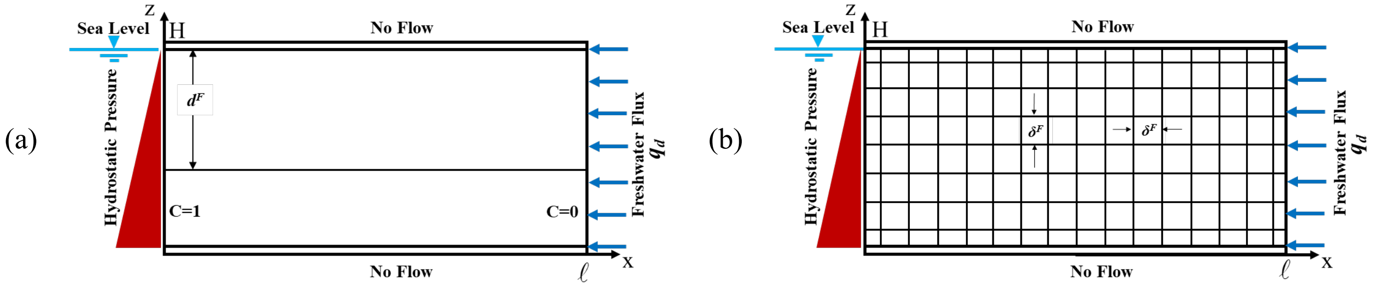

Fig. 1. Conceptual model of the fractured Henry Problem: (a) Single horizontal fracture configuration (SHF) and (b) Network of orthogonal fractures configuration (NOF).

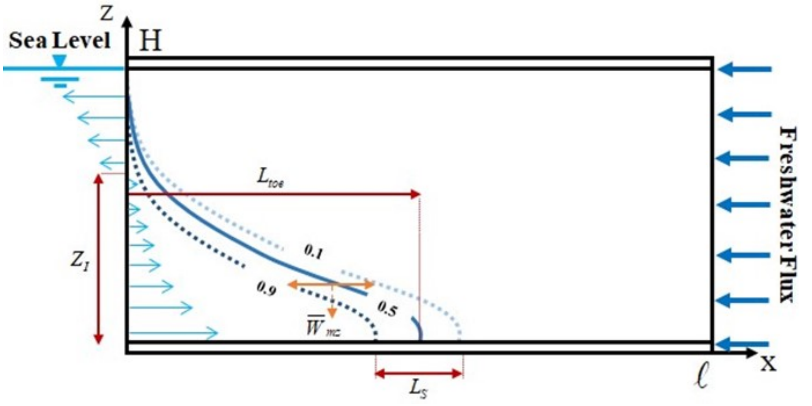

Fig. 2. Schematic representation of the SWI metrics (see [3])

2.3 Numerical model

The VDF-DFN simulations are performed using the finite element based software package of COMSOL Multiphysics®. The user-friendly environment of COMSOL, its ability to couple and solve various physical processes governing a problem, and also its post-processing features make it an ideal tool to be applied to various engineering applications. Our COMSOL model is created by adding and coupling “porous Media and Subsurface Flow” and “Transport of The Diluted Species” modules and by assuming concentration-dependent fluid density, expressed in Eq. (5). Examples of isochlors distribution obtained with the VDF-DFN model are depicted in Fig. 2a (SHF) and Fig. 2b (NOF).

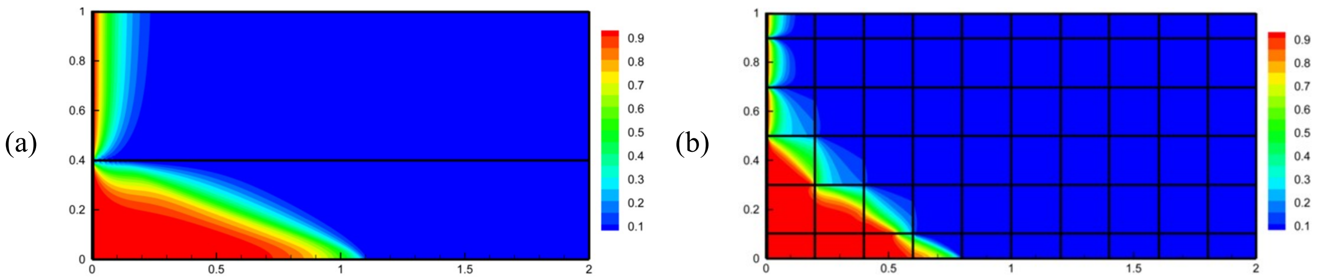

Fig. 3. Example of isochlors distribution obtained by COMSOL for: (a) SHF configuration and (b) NOF configuration (see fig 12 in [3]).

2.4 Global sensitivity analysis (GSA)

In this study, the variability of the model responses is quantified throughout a variance based technique using Sobol' Indices (SIs) as sensitivity metrics. To circumvent the problem of computational cost appearing because of the numerous evaluations of the model response, surrogate models, like the sparse PCE technic, have been proposed and used in the present study. The main idea is that each model output is expanded into a set of orthonormal multivariate polynomials whose maximum degree corresponds to the number of uncertain parameters. The polynomial coefficients are evaluated using the least-square technique that proceeds by minimizing an objective function representing the difference between the meta-model and the physical one. The great interest in such an approach is that SIs can be directly deduced from PCE coefficients. Hence, we calculated the first and total Sobol' Indices (SI) based on the PCE using multiple COMSOL outputs for each configuration. Univariate marginal effects are also calculated to obtain the global idea about the behavior of the model output in regards to the input parameter. Marginal effects of a certain parameter represent the variation of the model output when other parameters are kept constant at their average values. The readers can refer to Sudret (2008) [2], and Koohbor *et al.* (2018) [3] to get more details on GSA and PCE methods.

3 Results and Discussion

Among the numerous results obtained for this problem, the authors decided to present few which seem to have significant value to the literature. Fig. 4a shows the values of SIs for the length of the saltwater toe (L_{toe}) for SHF configuration. It is evident that the effect of d^F and K^F on the uncertainty of L_{toe} are most considerable among the rest of the parameters. It is also seen that L_{toe} is almost completely insensitive to $\alpha^F{}_{L,}$ which can be attributed to the ineffectiveness of the gravity on the horizontal fracture with a relatively small aperture. Fig. 4b shows the sensitivity of L_{toe} for NOF configuration. It is evident that the most uncertainty is generated because of K^F and δ^F. However, in this configuration K^F contributes the most to the sensitivity of L_{toe}. Also, it is worthy of mentioning that α^F in this configuration (NOF) contributes to some sensitivity of the model output; nevertheless, it is not the most significant input parameter.

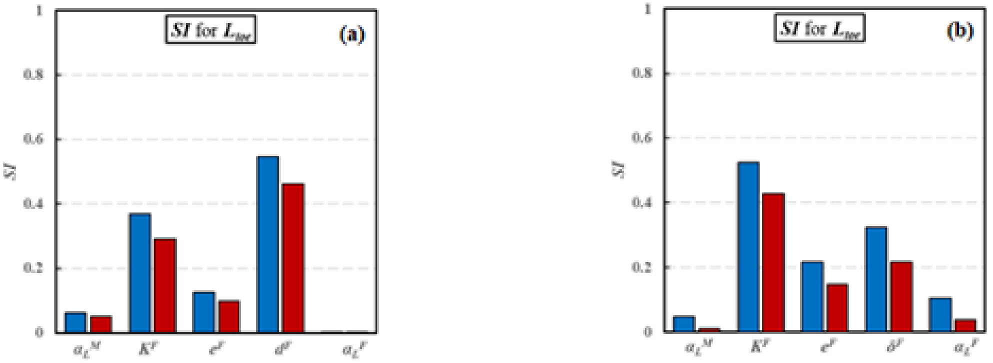

Fig. 4. Total (blue) and first order (red) SIs for L_{toe}: (a) SHF and (b) NOF

Fig. 5 gives the marginal effects of L_{toe} with respect to the variation of K^F as one of the input parameters that generates the most uncertainty. It is shown that for both configurations L_{toe} has a monotonic increase with respect to any increase of K^F. As for the SHF configuration (Fig. 5a), this is related to the fact that the increase of K^F concentrates the freshwater flow in the fracture and entails a weaker freshwater flow in the matrix. Consequently, the saltwater wedge expands landward and L_{toe} increases. The same explanation applies to NOF configuration (Fig. 5b) and is seen that the average rate of increase is even higher than SHF configuration and leads to a close to linear variation curve.

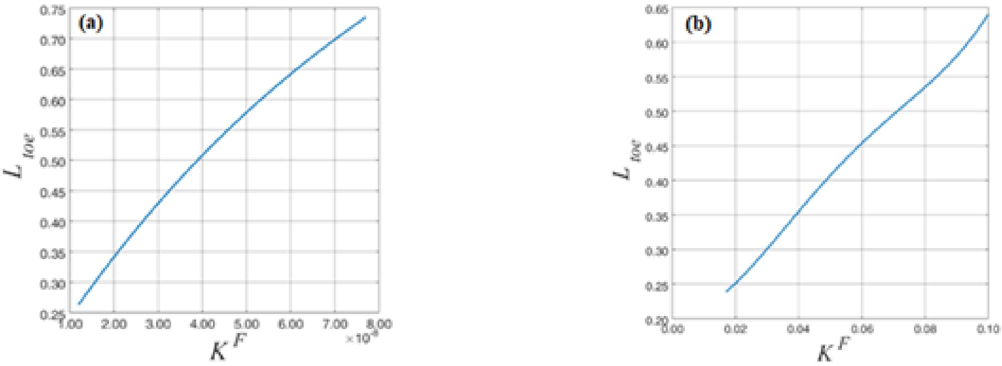

Fig. 5. The marginal effects of KF on L_{toe}: (a) SHF and (b) NOF

The sensitivity of the concentration distribution to K^F for both configurations is measured by calculating total SIs for individual cells in the domain (see Fig. 6). Fig. 6 shows that for both configurations the zones associated with largest SI for K^F are located within the saltwater wedge towards the low isochlors. In this region, the mass transfer is mainly governed by advection process. Advection process, mainly related to the velocity field, highly depends on fracture permeability. Although the general trend of sensitivity spatial distribution is similar for the two configurations, Fig. 6b shows a broader zone of influence suggesting more uncertainty associated with model output in regards to K^F for NOF configuration.

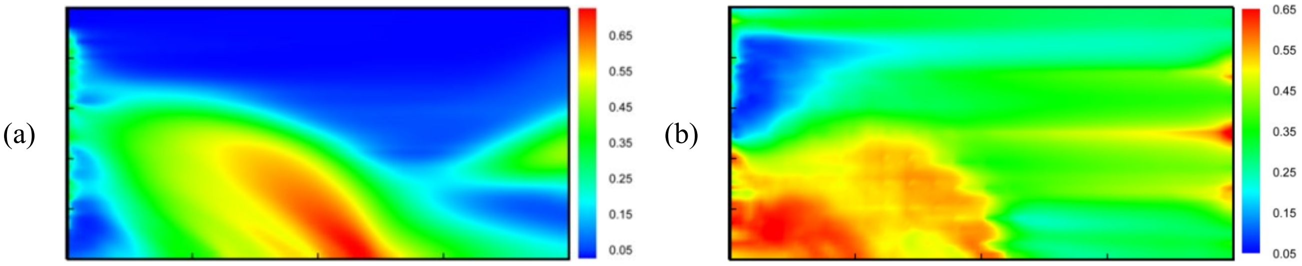

Fig. 6. GSA results for the spatial distribution of the salt concentration to K^F: (a) SHF and (b) NOF configuration

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