Adjoint-based control of a separated boundary-layer flow over a bump

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Résumé :

Une stratégie de contrôle optimal, capable de stabiliser un écoulement décollé bi-dimensionel au dessus d'une bosse vers son état d'équilibre est déterminée sur la base d'un seul actionneur. Ce dernier est piloté par une loi de commande, calculée par une méthode adjointe d'optimisation basée sur les équations de Navier-Stokes non linéaires. L'écoulement consideré est une couche limite décollée au dessus d'une bosse profilée. La dynamique instationnaire de cet écoulement est multi-échelle et compte des instabilités transitoires et auto-entretenues qui sont particulèrement difficiles à contrôler. De plus, l'écoulement introduit des délais variables entre l'actionneur et sa réponse et une modification significative de l'écoulement moyen par rapport à lécoulement de base connu sous nom de mode déplacement. Ces travaux montrent que le contrôle de ce mode est essentiel pour obtenir un contrôle optimal vers l'état d'équilibre. Cette idée est confirmée par la loi de commande optimale qui combine une stratégie d'aspiration décroissante en temps pour supprimer le mode, couplée à une stratégie de type jet pulsé pour atténuer les instabiltés.

Abstract :

The unstable flow over two-dimensional bump is controlled back to its steady state using a single actuator and a nonlinear adjoint-based approach for the control law. Here we consider a laminar boundary-layer flow over a shallow bump resulting in a long and thin recirculation bubble, subject to transient and self-excited instabilities. This flow configuration is particularly challenging for flow control algorithms : it is characterized by different types of instabilities with a broad range of time scales, the flow has long and variable time delays between the actuator, and the response of the flow and the baseflow and the meanflow differ substantially. We show that manipulating the shift between mean and baseflow is a key for succesfull control strategies. We demonstrate this idea combining a slowly decaying suction-type actuation together with a zero-net mass flux control strategy to achieve optimal control.

Mots clefs : Laminar flow, separated boundary layer, flow control.

1 Introduction

Flow control in laminar-separated flows such as bluff-body wakes or separation induced by an adverse pressure gradient over a flat-plate boundary layer remains a challenge for modern algorithms. Separation

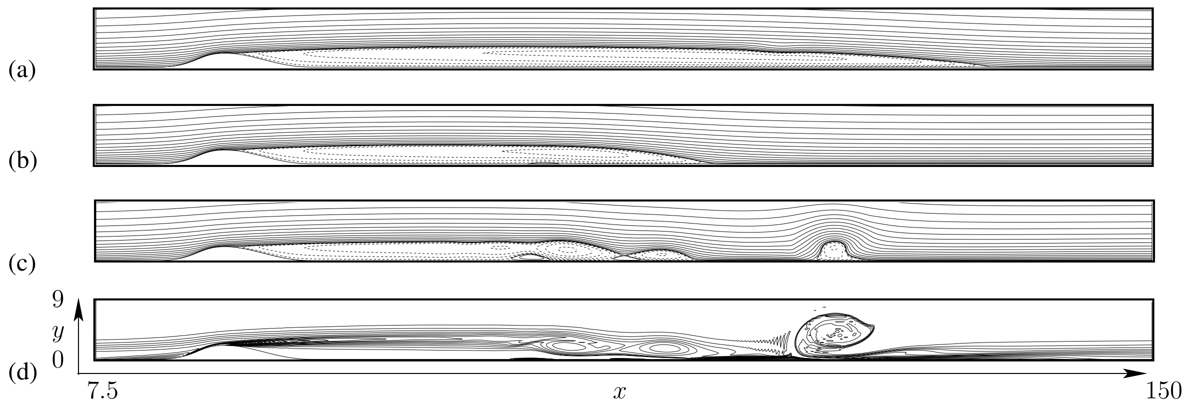

FIGURE 1 – Streamlines of the steady state $\mathbf{U}(x, y)$ (a), of the mean flow $\bar{\mathbf{u}}(x, y)$ (b) at $Re = 650$ and bump height $h/\delta = 2$. Snapshot of the streamlines (c) and of vorticity (d) of the fully developed flow for the same flow conditions.

induced by the geometry of the flow induces recirculation regions whose dynamics is not yet fully understood. Depending on their characteristics, recirculation regions can be subject to self-excited instabilities also known as resonator dynamics and transient growth associated with the amplification of perturbations. Recent progress has been made in modeling and control of either the amplified dynamics or the resonator dynamics [8] but control strategies capable of suppressing both dynamics simultaneously and drive the flow back to its steady state remain an open issue.

We address this problem using augmented Lagrangian optimization procedures to control a separated boundary layer over a bump. This particular flow geometry is known to exhibit a self-excited low-frequency flapping instability, characterized by a large scale oscillation of the recirculation region, while simultaneously amplifying perturbations localized upstream the separation region [4]. The control of this particular flow was already investigated using model reduction [5] and adjoint-based optimization methods [9]. However, neither of these approaches proved to be capable to control the flow back to its steady state and proved to lack robustness.

We circumvent this problem by controlling the baseflow and the instability dynamics using complementary approaches. Starting from a steady state forced by a suction actuator located near the separation point, the baseflow modification is shown to be controlled by a vanishing suction strategy. For weakly unstable flow regimes, this control law can be further optimized by means of direct-adjoint iterations of the nonlinear Navier-Stokes equations. In the absence of external noise, this novel approach proves to be capable of controlling the transient dynamics and the baseflow modification simultaneously. The present strategy allows of controlling the flow from a fully developed nonlinear state back to the steady state using a single actuator located at the separation point.

Finally we will introduce methods to compute control kernels using the adjoint method. The aim is to make the link between optimization and reduced-order controllers capable of performing feedback control.

2 Methods & Results

The two-dimensional Navier-Stokes equations, together with its adjoints are solved in a Cartesian domain where the bump is added in the domain using a change of variable described in [10]. Here the reference length scale is the displacement thickness of the incoming boundary layer δ and the Reynolds number is defined such that $Re = U\delta/\nu$. The non-dimensional height of the bump $h/\delta = 2$ produces long recirculation regions which become globally unstable for $Re \gtrsim 590$ [4]. The flow is shown in Fig. 1(a-c) at $Re = 650$, where vortex shedding is caused by the low-frequency flapping instability [11]. Details about the methods can be found in [10].

A blowing-suction actuator is introduced at the summit of the bump and we seek to find the optimal control strategy that minimizes the cost function

$$\begin{aligned} \mathcal{J}(\phi, \mathbf{u}') = \frac{1}{2} \int_{T_0}^{T_1} \int_{\Omega} \partial \mathbf{u}'/\partial t \cdot \partial \mathbf{u}'/\partial t \, \mathrm{d}\mathbf{x} \, \mathrm{d}t \\ + \gamma \int_{T_0}^{T_1} \int_{\Gamma_c} \mathbf{B}\phi \cdot \mathbf{B}\phi \, \mathrm{d}s \, \mathrm{d}t, \end{aligned} \tag{1}$$

where the perturbation is defined by $\mathbf{u}'(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}) - \mathbf{u}(\mathbf{x}, t)$. Here $\mathbf{u}$ is the instantaneous flow solution, $\mathbf{U}$ is the steady state shown in Fig. 1(a), $\mathbf{B}$ is the distribution of the blowing-actuator, Γ_c is the zone over which the control is applied at the wall while the time window over which the optimization is performed is $[T_0, T_1]$ and moved in time using receding time-horizon algorithm [2]. Instead of minimizing directly eq. (1) using the adjoint procedure described in [9], we use a different approach :

First, we consider the dynamics of the baseflow modification depicted in Fig. 1(a) and Fig. 1(b). More precisely, we compute the "slow" decay rate associated with the time necessary to drive the flow from the mean flow, shown in Fig. 1(b) and in Fig. 2(b) back to the steady state shown in Fig. 1(a). The sensibility of the baseflow modification with respect to the actuator located at the summit of the bump is then used to compute a control law $\phi(t)$ capable of driving the baseflow modification back to the steady state, thereby minimizing the cost function (1).

We then use the adjoint method to minimize the cost function (1) and update the control law $\phi(t)$ to suppress any "fast" transient modes (see Fig. 2) which may destabilize the flow during the mean flow control process.

The time evolution of the control strategy is illustrated in Fig. 3(a-g) by mean of snapshots of the vorticity. We start from a short recirculation bubble, obtained by continuous suction $\phi(t = 0) = -1.5 \times 10^{-2} U$ of the actuator shown in Fig. 3(a). The flow is then let to slowly evolve back to the steady state through a slow exponential decrease of the suction amplitude $\phi(t)$. As instabilities develop, $\phi(t)$ is updated to suppress these instability through the wave-canceling mechanism described in [7]. Note that at $t = 1800$, the flow has nearly reached its steady state and the instabilities are controlled through blowing and suction until the flow reaches the solution shown in Fig. 1(a).

3 Summary & Outlook

We propose a new control methodology using sensitivity analysis associated with baseflow modification, optimal baseflow control and Lagrangian-based optimization procedures for transient perturbation dynamics. We illustrate the method in the case of a separated boundary-layer flow over a bump and show that when starting the simulation far from the steady state, it is possible to control this globally unstable flow using a time vanishing suction strategy at the wall.

The present strategy takes advantage of the high sensitivity of the baseflow modification to suppress the instabilities and successfully drive the flow back to its steady state. The present approach leads to a simple

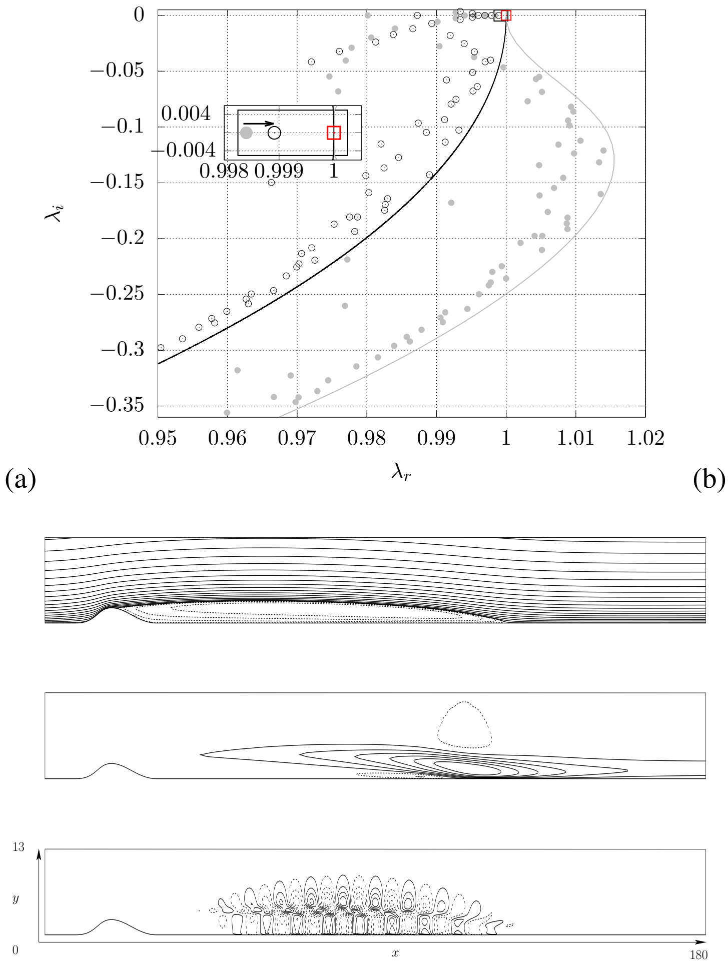

Figure 2 – (a) Lower half of the global spectrum computed using the SFD [3] together with the DMD [12] method showing the steady state (□), the associated modes (○) and the transformed modes (●). The continuous black line shows the marginal stability curve given by the unit circle. The continuous grey line shows the marginal stability of the stabilized Navier-Stokes equations using SFD for a damping coefficient of 0.0292 and a cutoff frequency of 14.99 [3]. The arrow shows the destabilizing effect of the SFD on the shift mode (see the insert), whereas the unsteady modes are effectively damped, below the marginal SFD curve (continuous grey line). (b) Streamlines associated with the steady state (□) (top), iso-contours of the streamwise velocity of the shift mode associated with the baseflow modification (middle) and iso-contours of the most streamwise velocity $\hat{u}$ amplified mode (●) (bottom).

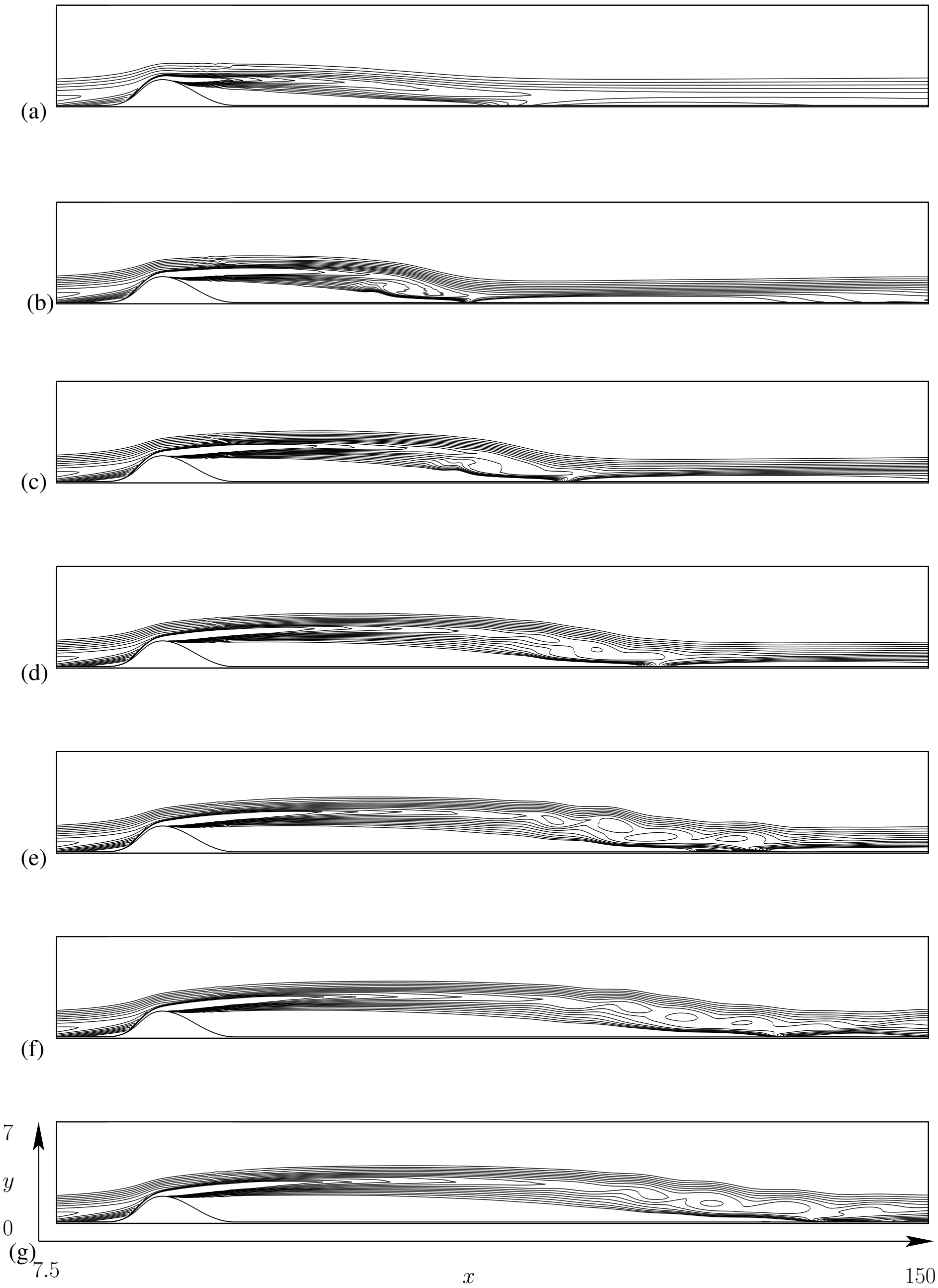

FIGURE 3 – Sequence of streamlines for the adjoint-based controlled strategy for $Re = 650$, $h/\delta = 2$ at (a) $T = 0$, (b) $T = 300$, (c) $T = 600$, (d) $T = 900$, (e) $T = 1200$, (f) $T = 1500$, and (g) $T = 1800$ for the terminal time control strategy. The y axis is stretched compared to the x axis.

solution strategy to manipulate the slowly decaying suction control that could be easily implemented in an experiment.

A recent study showed that the augmented Lagrangian approach could for instance be used to control the flow behind a cylinder [6] for values of the Reynolds number well beyond criticality. However the method requires very long time horizons for the optimization, which becomes numerically intractable for complex flow dynamics or three-dimensional configurations. The present approach, combining baseflow modification control and optimization using the augmented Lagrangian approach, could be a possibility to reduce the time horizons, which is the limiting factor in the case of complex three-dimensional applications for flow optimization algorithms.

We are currently exploring solutions to compute control kernels based on the present method. The aim is to determine the time-dependent optimal control gain resulting from the present optimization. This can be achieved using minor modifications of adjoint methods [1] and could provide a matrix-free approach to laminar flow control.

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