INFLUENCE OF RHEOLOGICAL BEHAVIOR OF ELASTO-HYDRODYNAMIC POROUS SELF-LUBRICATING JOURNAL BEARINGS

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Abstract:

The Elasto-hydrodynamic lubrication of finite porous self-lubricating journal bearings is investigated taking into account the rheological lubricant behavior effect. The modified Reynolds equation is derived by considering both the fluid flow in the porous matrix and the lubricant rheological behavior where Darcy’s law and power-law model were used. Governing differential equations were solved numerically using the finite difference method. Obtained results showed that the power law index, \( n \), has important effects on the performance of porous and non-porous elastic journal bearings. An improvement in the fluid flow bearing characteristics (load capacity, pressure) is observed for dilatant fluids compared to Newtonian fluids. The permeability of the porous structure has significant effects on the performance of porous journal bearings of finite length, particularly at higher eccentricity ratios. It is demonstrated that the elastic deformation decreases the load carrying capacity.

Keywords: Elasto-hydrodynamic lubrication; Porous journal bearing; Non-Newtonian fluid; Power law.

Nomenclature

\begin{itemize}
  \item \( C \) \quad Radial clearance [m]
  \item \( C_0 \) \quad Flexibility parameter
  \item \( e \) \quad Eccentricity [m]
  \item \( D \) \quad Bearing diameter [m]
  \item \( h \) \quad Film thickness [m]
  \item \( K \) \quad Permeability [m\(^2\)]
  \item \( L \) \quad Length bearing [m]
  \item \( m \) \quad Power law consistency coefficient [Pa.s\(^n\)]
  \item \( n \) \quad Power law index
\end{itemize}
Introduction

Porous journal bearings are used in devices and general machinery are made of a porous bush filled with lubricating oil. Consequently, the bearings require no further lubrication during the whole life of the machinery. Their self-lubricating properties and low cost give them the advantage to be used in areas where the conventional bearings are unusable such as in situations of congestion and inaccessibility for lubrication. That is why the choice of lubricants with high lubricating power is essential. The rheological behavior of lubricants has shown that the Newtonian hypothesis of lubricants is far from the physical reality. Deformation is inevitable in journal bearing lubrication. It becomes important in some cases, especially when their size is considerable comparing to the size of the film thickness [1].

Many theoretical and experimental studies on porous journal bearings showed that the permeability of porous bush reduces pressure and load capacity and increases in the friction coefficient [2-3]. However, the flexibility of the porous liner was ignored in these studies and the lubricant is considered as Newtonian fluid. Mokheimar et al. [4] studied the elastic deformations of the solid liner (rigid bush case). Their results showed that journal bearings performances decrease when elastic deformation increases. Their theoretical model was used by Lin et al. [5] to calculate the static characteristic of long and flexible porous journal bearing. The behavior of porous bearing with finite length taking into account the elastic deformation has been studied by Elsharkawy et al. [3] using Brinkman-Darcy extended model. Notice, in all these studies, the non-Newtonian effect has been neglected.

Moreover, in many engineering applications, it is demonstrated successfully [6-8] that the fluid lubricants can be described by the power law model such as pseudo-plastic (n<1) and dilatant (n>1) fluids. Ju et al. [9] analyzed the thermo-hydrodynamic aspect of a finite width hydrodynamic journal
bearing using non-Newtonian lubricants. The used lubricants are described by the power-law model. The authors showed that the thermal effects are more pronounced at higher values of power law index. Malki et al. [10] were interested to calculate numerically the porous journal bearing characteristics taking into account the power law of the lubricant fluid. Their results confirm that dilatant fluids increase the performance of bearings and can recover the degradation caused by the presence of the porous matrix. In the present work, a numerical simulation is performed for elasto-hydrodynamic self-lubrication aspect analysis in the porous circular journal bearing of finite length with sealed ends. The journal bearing characteristics are calculated taking into account the elastic deformation in the porous matrix and rheological behavior of the lubricant.

2 Physical and mathematical models

2.1 Geometrical configuration

Fig. 1 show the physical configuration of the porous journal bearing considered in this study. The journal rotates with a constant angular velocity around its axis. By considering the shaft and the porous bush as aligned, we can calculate the position of the shaft in the bush. The film thickness taking into account the elastic deformation, δ, of the bearing liner is given by [4]:

\[ h = C (1 + \varepsilon \cos \theta) + \delta \]  

\[ C_0 = \frac{\mu_0 \omega R^2 (R_e - R_c)(1 - \nu^2)}{C^3 E} = \frac{\delta}{p} \]  

Fig. 1. Schematic representation of a porous journal bearing.

2.2 Rheological model

The expression of the generalized viscosity for an incompressible fluid in the power-law model [10] is:

\[ \mu = m (2 \Pi)^{\frac{n-1}{2}} \]  

\( \Pi \), is the second invariant of the deformation rate tensor given by:

\[ \Pi = \sum \sum \varepsilon_{ij} \varepsilon_{ji} \]  

n : is power law index.
We can distinguish three types of fluids:

- For: \( n=1 \), the fluid behavior is Newtonian.
- For: \( n<1 \), the fluid behavior is pseudo-plastic.
- For: \( n>1 \), the fluid behavior is dilatant.

### 2.3 Mathematical model

Consider the geometric and velocity conditions and the interaction effects between the fluid film and the porous bush using Darcy’s law. The generalized Reynolds equation for porous journal bearing taking into account the viscosity variation along the film thickness [10] is given by

\[
\frac{\partial}{\partial x} \left[ \left( G^+ \frac{K F}{\mu} \right) \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \left( G^+ \frac{K F}{\mu} \right) \frac{\partial P}{\partial z} \right] = \omega R_s \frac{\partial}{\partial x} \left( \rho h - F - \frac{K}{\mu} \frac{\partial P^*}{\partial R} \right) \bigg|_{R=R_b} \tag{5}
\]

Where:

\[
G = \int_0^H \frac{R y}{\mu} dy - I_1 F; \quad F = \frac{1}{l_2} \int_0^H \frac{y}{\mu} dy; \quad I_2 = \int_0^H \frac{y}{\mu} dy; \quad J = \int_0^H \frac{dy}{\mu} \tag{6}
\]

Combining the porous expression velocity with the continuity equation, we obtain Darcy’s law characterizing the fluid flow in the porous matrix.

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( \frac{R}{\mu_b} \frac{\partial P^*}{\partial R} \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\mu_b} \frac{\partial P^*}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu_b} \frac{\partial P^*}{\partial z} \right) = 0 \tag{7}
\]

The components, \( u \) and \( w \), of the fluid film velocity are:

\[
\begin{align*}
    u &= \frac{\partial P}{\partial x} \left( 1 - J F \right) + \frac{u_2 - u_1}{l_2} J + u_1 \\
    w &= \frac{\partial P}{\partial z} \left( 1 - J F \right) + \frac{w_2 - w_1}{l_2} J + w_1
\end{align*} \tag{8}
\]

Where:

\[
I = \int_0^y \frac{\xi}{\mu} d\xi, \quad J = \int_0^y \frac{d\xi}{\mu} \tag{9}
\]

The conditions at interfaces and boundary conditions on the pressure are:

\[
\begin{align*}
    p\left( \theta, \frac{L}{2} \right) &= 0 \\
    p(\theta_c) &= 0; \quad \frac{\partial P}{\partial \theta} (\theta_c) = 0 \\
    p(0, z) &= P(2\pi, z) \tag{10}
\end{align*}
\]

\[
\begin{align*}
    P^*(\theta, R_b, z) &= p(\theta, z) \tag{11}
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial P^*}{\partial z} \left( \theta, R, \frac{L}{2} \right) &= 0 \\
    \frac{\partial P^*}{\partial R} \left( \theta, R_e, z \right) &= 0 \\
    P^*(0, R, z) &= P^*(2\pi, R, z) \tag{12}
\end{align*}
\]
The load capacity and the attitude angle are obtained by integrating the pressure field on the surface of the journal bearing. At equilibrium, we have:

\[ \bar{Q} = \frac{Q}{\mu_0 \omega R_0^2 L/C^2} = \sqrt{\frac{\bar{Q}_L}{2\pi}} + \frac{\bar{Q}_K}{2\pi} \]  

(13)

\[ \bar{Q}_L = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \bar{P} \sin \theta \, d\theta \, d\bar{z} \]  

(14)

\[ \bar{Q}_K = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \bar{P} \cos \theta \, d\theta \, d\bar{z} \]  

(15)

\[ \tan \phi = -\frac{\bar{Q}_L}{\bar{Q}_K} \]  

(16)

### 3 Numerical procedure

The previous differential equations governing the fluid flow in the porous bush and fluid film are of elliptic type. Their resolution is done with an iterative finite difference scheme including elastic deformation effects and non-Newtonian behavior of lubricants. Static performances of the porous journal bearing of finite length are obtained using a computational method which consists to initialize different variables such as: pressure, velocity, viscosity and deformation. The numerical scheme is performed iteratively for the determination of pressure in the porous matrix and fluid film, the viscosity and the film thickness. The Gauss Seidel method with an over-relaxation factor is used to solve these algebraic equations. For all these parameters, the iterative method is stopped when the test of convergence criteria is verified. This test is given by:

\[ \left| \frac{p^{\text{new}} - p^{\text{old}}}{p^{\text{old}}} \right| \leq 10^{-4} \]  

(17)

\[ \left| \frac{\mu^{\text{new}} - \mu^{\text{old}}}{\mu^{\text{old}}} \right| \leq 10^{-4} \]  

(18)

\[ \left| \frac{\delta^{\text{new}} - \delta^{\text{old}}}{\delta^{\text{old}}} \right| \leq 10^{-4} \]  

(19)

### 4 Results and discussions

Table 1 gives the porous journal bearing technical data used in the numerical simulation.

<table>
<thead>
<tr>
<th>Shaft diameter, (D_s)</th>
<th>100 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>External bush diameter, (D_{eb})</td>
<td>150 mm</td>
</tr>
<tr>
<td>Length bearing, (L)</td>
<td>100 mm</td>
</tr>
<tr>
<td>Radial clearance, (C)</td>
<td>100 µm</td>
</tr>
<tr>
<td>Revolution speed</td>
<td>1000 rev/min</td>
</tr>
<tr>
<td>Relative eccentricity, (e/C)</td>
<td>0.7</td>
</tr>
<tr>
<td>Dimensionless permeability, (K)</td>
<td>(10^{-5} - 10^{-1})</td>
</tr>
<tr>
<td>Power law index, (n)</td>
<td>0.9, 1, 1.1</td>
</tr>
<tr>
<td>Elastic deformation parameter, (C_0)</td>
<td>0, 0.04</td>
</tr>
</tbody>
</table>
Fig. 2 represents the dimensionless pressure variation of the lubricating fluid film in the median plane position of the bearing versus the circumferential coordinate for dimensionless permeability equal $10^{-2}$. The figure shows the existence of a pressure, concentrated in the middle position of the bearing and corresponding to an angular position of 160°. It should be noted that the increase in the power law index, $n$, generates a considerable hydrodynamic pressure. The pressure decreases with the increase of elastic deformation parameter.

![Graph showing pressure variation](image1)

**Fig. 2.** Circumferential pressure evolution versus power law index and elastic deformation parameters.

![Graph showing load capacity evolution](image2)

**Fig. 3.** Evolution of load capacity versus permeability for different values of power law index and elastic deformation parameters.
Fig. 3 illustrates the elastic deformation effect on the load carrying capacity of the porous journal bearing for three values of power law index versus dimensionless permeability. Fig. 3 shows a decrease in the load carrying capacity with an increase of permeability and an elastic deformation parameter. It can be seen that the dimensionless load capacity with a power law index value, $n=1.1$, is greater than a Newtonian case.

![Fig. 3](image)

**Fig. 4.** Evolution of attitude angle versus permeability for different values of power law index and elastic deformation parameters.

![Fig. 4](image)

**Fig. 5.** Variation of the friction coefficient versus permeability for different values of power law index and elastic deformation parameters.
The evolution of attitude angle versus permeability for different values of power law index and elastic deformation parameters is shown in Fig. 4. The attitude angle is inversely proportional to the power law index and elastic deformation parameters. In addition, the attitude angle increases with increasing permeability from approximately a critical value of $10^{-3}$.

The friction coefficient is defined as the ratio of the friction force to the hydrodynamic load capacity. The evolution of this coefficient versus permeability for different values of power law index and elastic deformation parameters is displayed in Fig. 5. It can be noted that the friction coefficient decreases with the increase in the power law index, $n$. Notice that the friction parameter is proportional to the permeability.

## 5 Conclusions

The effect of elastic deformation parameter on static characteristics of finite porous journal bearings with seals by taking into account rheological behavior of lubricants is studied numerically. The influence of the porous matrix (permeability), the rheological behavior of lubricant ($n$), and the elastic deformation of the bearing are analyzed. Obtained results showed:

- The pressure, the load carrying capacity and the attitude angle decrease with the increase of the deformation parameter $C_0$. The decrease is more pronounced for high permeability cases.
- The effect of the porous facing on the bearing surface decreases the pressure and load carrying capacity but increases the attitude angle and friction coefficient.
- The power law index, $n$, has important effects on the performance of porous journal bearings. The use of dilatant fluids increases their static performances.

## References