Cylinders Calculated using the Generalized Beam Theory

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Résumé

Although representing the simplest shell system, cylindrical shells still resist a closed form solution. Among the first researchers to attempt an analytical description for thin cylindrical shells under bending was Brazier. He recognized that the classical St. Venant theory would not be able to account for the large cross section distortion, typically the ovalization. Donnell presented a first set of coupled non-linear differential equations, which, since then, is often taken as a reference and basis for further extensions. Donnell's theory includes second order terms necessary to formulate the stability; it was used for decades to estimate the buckling load of cylinders by linearization of the second order terms. Around the same time Flügge came up with his Theory, commonly named after him, which also allowed for similar calculations. Due to some constraining assumptions in the theories, Donnell's and Flügge's formulas deliver good results in ranges complementary of cylinders lengths: while Donnell's theory seems to better perform for "long" cylinders Flügge's better covers the "short" ones. Further improvements to overcome some simplification in Donnell's work were made from Morley, and later by Simmonds, Koiter and Morley.

A major improvement to a mechanically accurate description of cylindrical shells was achieved by Schardt by applying the principles of the Generalized Beam Theory (GBT). The GBT foots on earlier works of Vlassov and represents a comprehensive beam theory that overcomes the limitations of Bernoulli's assumption by taking into account non-uniform axial out-of-plane displacements. Thus conventional theories for the analysis of prismatic thin-walled structural members were unified and extended by the GBT. Indeed, the central element of the GBT is the concept of warping functions where, with k modes of deformation associated with a corresponding pattern of cross-sectional displacements kû, axial and shear stresses as well as corresponding section properties (kC, kD and kB). The keystone of the GBT is that all these warping functions are made orthogonal to each other by appropriate mathematical transformations. Schardt initially formulated the GTB for the general case of open profiles, but extended it to closed ones, and more specifically to cylindrical shells. This publication will deal with the basics of the method, considering first and second order theory for the case of isotropic material and extending the classical assumptions made in shell theory (i.e. no shear and no radial membrane deformations).

Mots-Clés: cylindre, shell, mechanics, general, beam, theory

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