Inviscid stability of finite-amplitude sand dunes

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Résumé

Sand ripples and dunes, as observed in the natural environment or in the industry, result from the interaction of an erodible ground with a liquid or gas flow. Their formation is usually understood as the result of a linear instability mechanism, where the competition of destabilizing fluid inertia and stabilizing gravity and particle relaxation phenomena, leads to the emergence of some most amplified bed disturbance. Accordingly, the predicted most amplified eigenmode is necessarily sinusoidal in space. However, observations of sinusoidal bedforms are scarce, even at the early stages of their growth. Most experiments rather exhibit the development of sawtooth-like patterns. Under water, these bed features grow from randomly distributed small protuberances and very soon exhibit a sharp rim. This rim most likely causes, and may be caused by, separation of the fluid stream. Aeolian ripples, although they arise from a quite different mechanism involving saltating grains, also display early coarsening, associated with flow separation and the development of sharp edges.
Flow separation over dunes thus appears as a major feature, whose importance in the selection of the dune shape and size has as yet received little attention. Flow separation past an obstacle is a partially open topic in itself, which arises from a complex interaction between a (possibly turbulent) boundary layer and the outer inviscid flow. A potential flow solution was found by Helmholtz and Kirchhoff, where "free streamlines" issued from the separation points enclose a "wake" of dead fluid. A practical conformal-mapping transformation to calculate the Kirchhoff-Helmholtz flow past rounded obstacles was found by Levi-Civita (1907). However, the position of the separation point remained free. Whereas for a solid surface with a corner it came natural (what later became the Kutta-Joukovsky condition) to attach the separation streamline at the corner itself, the position of separation over a smooth surface remained undetermined. It was later observed by Brillouin (1911) and Villat (1914) that the free streamline is generally singular at the separation point, except for a distinguished position along the obstacle – today known as the Brillouin-Villat point – where the singular term vanishes and the curvature and pressure gradient remain finite.

For the erodible dunes considered here, their shape results from the interaction of the fluid flow with the thin sand layer moving at the dune surface. A brink is present but its position is a priori unknown. This indetermination is similar to that of the position of the separation point on a rounded fixed obstacle. The purpose of this communication is to show that the Brillouin-Villat criterion may be adopted in order to close the problem and produce a self-maintaining sharp-edged dune shape. Within the sand flux model used here, where the bed shear-stress is in phase with the flow velocity, the flat bed is linearly stable, so that it is the

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separation itself which allows the existence of a self-preserving dune shape.
Emphasis will be put on dunes travelling without deformation on a non-erodible ground, where the Brillouin-Villat condition also provides the dune velocity. It will be shown that the selected profile is stable against small perturbations of the separation point. The results and their relevance regarding experiments will be finally discussed, as well as the limits of the inviscid theory.

Mots-Clés: Inviscid stability, flow separation, finite, amplitude sand dunes