Falling liquid films in interaction with a confined counter-current gas

G. LAVALLE[a,c], Y. LI[b], S. MERGUI[b], N. GRENIER[a], G. F. DIETZE[b]

a. Laboratoire LIMSI, CNRS, Univ. Paris-Sud, Université Paris-Saclay, F-91405, Orsay, France (gianluca.lavalle gmail.com)

b. Laboratoire FAST, Univ. Paris-Sud, CNRS, Université Paris-Saclay, F-91405, Orsay, France

c. Present address: IMFT, Institut de Mécanique des Fluides de Toulouse, 31400 Toulouse, France

Abstract:

We aim at studying the stability and the dynamics of liquid films sheared by a confined counter-current gas flow within vertical and inclined channels. Such a two-phase flow is commonly employed in several engineering applications, such as distillation processes. The nature of the interfacial dynamics is the key to intensify heat and mass transfer between the liquid and the gas: the more the interface is disturbed by waves, the greater the inter-phase transfer is. This is valid as long as the liquid does not flood the channel, which represents an unwanted situation for the industrial processes.

In this work, we focus firstly on the linear stability of such a two-phase flow. We show that the Kapitza instability, responsible for the development of long waves on the liquid film, can be fully suppressed by confining the gas phase with an upper wall. This is valid in both the configurations of counter-current laminar gas and aerostatic gas, i.e. when the pressure gradient balances the gravity in the gas, which displays a Couette-like velocity profile. The critical confinement at which the suppression of the Kapitza instability occurs depends on the angle of inclination of the channel as well as on the liquid Reynolds number.

At weaker confinements instead, the cut-off wavenumber displays a non-monotonic trend by increasing the gas velocity, meaning that the imposed gas flow rate is able to control the transition between stable and unstable regions in falling films. These results have been obtained by means of the numerical resolution of the temporal two-phase Orr-Sommerfeld problem, and have been also validated by our experimental observations.

In a second step, we study the non-linear interfacial dynamics in the configuration of gas-liquid flows in confined inclined and vertical channels. For this, we employ both direct numerical simulations and a two-phase long-wave integral model. At low inclination angles and under aerostatic gas conditions, increasing the level of confinement provides a non-monotonic behaviour of the maximum film thickness, i.e. the crest of travelling waves. By increasing instead the velocity of the counter-current gas flow at very strong confinements, the maximum film thickness simply decreases. This occurs both at low inclination angles and in vertical channels, although for the latter case the effect of the gas flow is less pronounced.

Keywords: falling films, gas flow, Kapitza instability

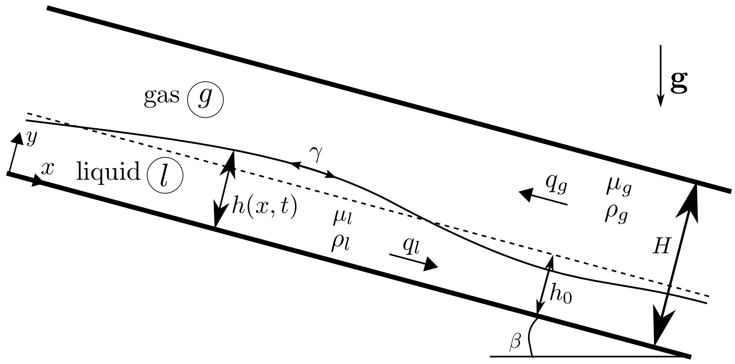

Figure 1: Sketch of the considered problem: a liquid film falling down an inclined wall whilst interacting with a strongly-confined gas phase. The gas is either subject to an aerostatic pressure difference or flows counter-currently at an imposed flow rate.

Introduction

We consider a liquid film flowing in an inclined channel and in interaction with a confined gas flow, as shown in Figure 1. The gas can be either aerostatic, i.e. driven by a pressure difference which balances the gravity, or flowing counter-currently.

We revisit the linear stability of this two-phase flow by numerically investigating the effect of the gas on the convective long-wave interfacial mode (Kapitza instability [5]) in the case of strong confinement. We also study how the non-linear interfacial dynamics is affected by the confinement of the gas phase.

The governing equations

With the aim to study the linear stability of the two-phase flow of Figure 1, we perform a temporal stability analysis by employing the well-known two-phase Orr-Sommerfeld equations. The linear problem is identical to the problem studied by Tilley et al. [7]. By introducing the stream-function perturbations $\phi^\star$ in the liquid phase and $\psi^\star$ in the gas phase ($\hat{\phi}$ and $\hat{\psi}$ denote the amplitudes of the corresponding perturbations), the probleam reads:

$$\hat{\phi}'''' - 2\alpha^2\hat{\phi}'' + \alpha^4\hat{\phi} = i\alpha[(U_l - c)(\hat{\phi}'' - \alpha^2\hat{\phi}) - U_l''\hat{\phi}], \tag{1}$$
$$\hat{\psi}'''' - 2\alpha^2\hat{\psi}'' + \alpha^4\hat{\psi} = i\alpha[(U_g - c)(\hat{\psi}'' - \alpha^2\hat{\psi}) - U_g''\hat{\psi}], \tag{2}$$

subject to boundary conditions at the wall:

$$\hat{\phi}(0) = 0, \quad \hat{\phi}'(0) = 0, \tag{3}$$
$$\hat{\psi}(H) = 0, \quad \hat{\psi}'(H) = 0, \tag{4}$$

and at the interface:

$$\hat{\phi}' + U_l'\hat{\phi}\underline{c}^{-1} = \hat{\psi}' + U_g'\hat{\phi}\underline{c}^{-1}, \tag{5}$$
$$\hat{\phi}'' + U_l''\hat{\phi}\underline{c}^{-1} + \alpha^2\hat{\phi} = \Pi_\mu[\hat{\psi}'' + U_g''\hat{\phi}\underline{c}^{-1} + \alpha^2\hat{\psi}], \tag{6}$$
$$2\alpha^2\hat{\phi}' - i\alpha\Pi_\rho[\hat{p}_l + P_l'\hat{\phi}\underline{c}^{-1}] + We\, i\alpha^3\hat{\phi}\underline{c}^{-1} = 2\alpha^2\Pi_\mu\hat{\psi}' - i\alpha\Pi_\rho[\hat{p}_g + P_g'\hat{\phi}\underline{c}^{-1}], \tag{7}$$

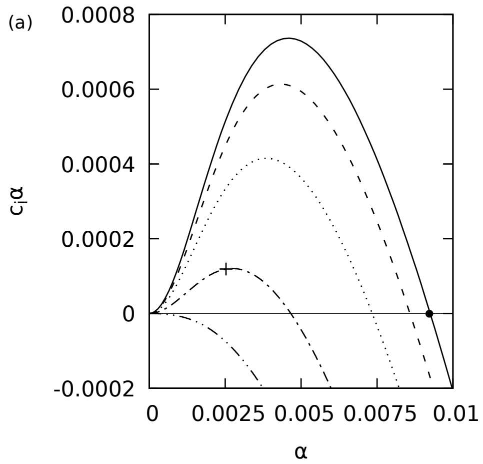

Figure 2: Influence of an increasingly strong counter-current air flow on the stability of a falling water film at $\beta = 3°$, $Re_l = 23.9$ and $\eta = 3.6$. Temporal growth rate. Solid line: $\Delta p = \Delta p^a$ ($Re_g = 3.0$); dashed line: $\Delta p = 10\Delta p^a$ ($Re_g = -0.94$); dotted line: $\Delta p = 25\Delta p^a$ ($Re_g = -7.2$); dash-dotted line: $\Delta p = 50\Delta p^a$ ($Re_g = -17.1$); dash-dotted-dotted line: $\Delta p = 70\Delta p^a$ ($Re_g = -24.4$). The filled circle marks the cut-off wavenumber α_c^a of the aerostatic situation.

where α is the real wavenumber and $c = c_r + ic_i$ the complex wave celerity, while p and U the pressure and the velocity. The Reynolds number is defined as $Re = \mathcal{UL}/\nu$, the Weber number as $We = \gamma(\rho_l \mathcal{L}\mathcal{U}^2)^{-1}$, where γ is the surface tension, $\mathcal{L} = \nu_l^{2/3} g^{-1/3}$ and $\mathcal{U} = (\nu_l g)^{1/3}$ are reference scales. The pressure perturbation amplitudes $\hat{p}_l$ and $\hat{p}_g$ in eq. (7), evaluated at $y = h_0$, can be recovered directly from the Navier-Stokes equations (Yih [9]):

$$\hat{p}_l = \Pi_\rho^{-1}[\underline{c}\hat{\phi}' + U_l'\hat{\phi}] + (i\alpha\Pi_\rho)^{-1}(\hat{\phi}''' - \alpha^2\hat{\phi}'), \quad (8)$$

$$\hat{p}_g = [\underline{c}\hat{\psi}' + U_g'\hat{\psi}] + \Pi_\mu(i\alpha\Pi_\rho)^{-1}(\hat{\psi}''' - \alpha^2\hat{\psi}'), \quad (9)$$

where $\Pi_\mu = \mu_g/\mu_l$, $\Pi_\rho = \rho_g/\rho_l$ and $\underline{c} = c - U|_{h_0}$. The system (1-9) is solved by means of continuation using the software AUTO-07p (Doedel [4]). This code was previously used in Dietze & Ruyer-Quil [2]. In our analysis, we will vary the relative confinement of the film, which is defined as $\eta = H/h_0$, where H is the channel height and h_0 the Nusselt equilibrium film thickness. It constitutes one of the control parameters of the problem, together with the inclination angle β and the liquid and gas flow rates q_l and q_g.

Results

In this section, we show the effect of a counter-current gas flow upon the linear stability of the liquid film. We particularly show that the gas flow can stabilize the liquid film up to the point of fully suppressing the Kapitza instability. This is shown in Figure 2, where the temporal growth rate for a relatively strong confinement ($\eta = 3.6$) and for the strongest gas flow rate ($Re_g = -24.4$) is negative for all wavenumbers. The suppression is caused by the high level of confinement. It is the tangential viscous stress exerted by the gas on the film surface that plays a decisive role in the stabilization and the suppression (Lavalle et al. [6]). There is a phase shift of almost exactly half a wavelength between the film thickness perturbation and the associated shear stress perturbation. Consequently, the adverse shear stress is greater at a wave hump than at a wave trough. This tends to homogenize the local equilibrium flow rate within the liquid

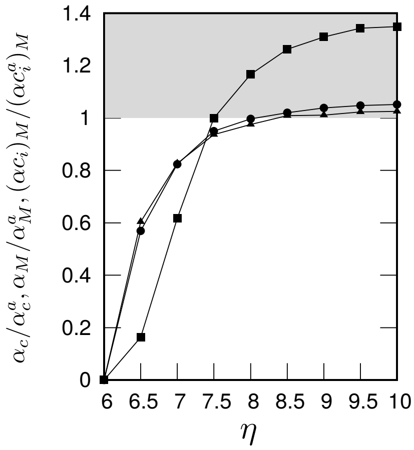

Figure 3: Effect of the relative confinement η on the stability of a falling water film at $\beta = 3°$, $Re_l = 17$ and $Re_g = -67.5$. Deviation of the cut-off wavenumber α_c (filled circles), most unstable wavenumber α_M (filled triangles) and maximum growth rate $(\alpha c_i)_M$ (filled squares) from their respective aerostatic limits. The grey area marks the unstable region compared to the aerostatic scenario.

film, thus weakening the inertia-driven mechanism of the Kapitza instability (Dietze [3]).
At lower gas flow rates than the critical value corresponding to the suppression, the cut-off wavenumber decreases for all wavenumbers w.r.t. the limit of aerostatic gas (Figure 2).

However, this is valid only at relatively strong confinements. Indeed, at weaker confinements, the gas flow destabilizes the liquid films. The destabilization provided by a counter-current gas flow was also observed in previous works (Trifonov [8]), although the gas flow in our case is laminar and not turbulent. Figure 3 shows, for an imposed gas Reynolds number, how the cut-off wavenumber, the most unstable wavenumber and the maximum growth rate depend on the relative confinement η, which is varied in the range $(6, 10)$. One observes that there exists a threshold value for the confinement (around $\eta = 7.5$) above which the gas has a destabilizing effect. The most unstable and the cut-off wavenumber follow a similar trend and vary smoothly at weaker confinements ($\eta > 8$). The maximum growth rate has a sharper behaviour than the most unstable and cut-off wavenumber: it decreases more in the stabilizing region and increases more in the destabilizing region. Interestingly, there exist some values of relative confinement ($\eta = 7.5 - 8$) for which the growth rate is greater than the aerostatic limit while the cut-off wavenumber remains smaller than the corresponding aerostatic value. This was also observed experimentally by Alekseenko [1] for turbulent gas flows.

Conclusions

We have shown the effect of a counter-current gas flow upon the stability of a falling film in an inclined confined channel. By employing a temporal analysis of the linearized Orr-Sommerfeld problem, we have observed that the gas can fully suppress the Kapitza instability at strong confinement and this is due to the phase shift between the tangential viscous stress and film thickness perturbations. We have also identified the presence of a confinement threshold value above which the gas flow destabilizes the liquid film.

We conclude by discussing the effect of the gas flow on the non-linear interfacial dynamics. The trend is analogous to the linear dynamics. Indeed, Figure 4 shows, at relatively strong confinement, that a counter-current gas flow reduces the amplitude of saturated non-linear interfacial waves.

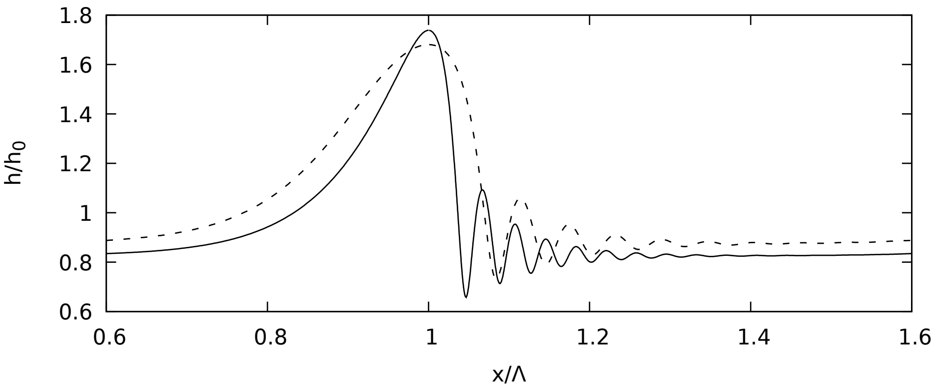

Figure 4: Influence of an increasingly strong counter-current air flow (the gas flow increases from solid line to dashed line) on the interfacial non-linear gas-liquid dynamics.

References

[1] Alekseenko S. V., Aktershev S. P., Cherdantsev A. V., Kharlamov S. M., Markovich D. M., Primary instabilities of liquid film flow sheared by turbulent gas stream, Intl. J. Multiph. Flow, 35 (2009).

[2] Dietze G. F., Ruyer-Quil C., Wavy liquid films in interaction with a confined laminar gas flow, J. Fluid Mech., 722 (2013).

[3] Dietze G. F., On the Kapitza instability and the generation of capillary waves, J. Fluid Mech., 789 (2016).

[4] Doedel E. J., AUTO-07p: Continuation and Bifurcation Software for Ordinary Differential Equations. Montreal Concordia University, 2008.

[5] Kapitza, P. L. Wave flow of thin layers of a viscous fluid (in Russian), Zh. Eksp. Teor. Fiz., 18 (1948).

[6] Lavalle G., Li Y., Mergui S., Grenier N., Dietze G. F., Suppression of the Kapitza instability in confined falling liquid films, J. Fluid Mech., 860 (2019).

[7] Tilley, B. S., Davis, S. H., Bankoff, S. G. Linear stability theory of two-layer fluid flow in an inclined channel. Phys. Fluids, 6 (1994).

[8] Trifonov Y. Y., Linear stability theory of two-layer fluid flow in an inclined channel, Intl. J. Multiph. Flow, 95 (2017).

[9] Yih C. S., Instability due to viscosity stratification, J. Fluid Mech., 27 (1967).