Numerical study of forced, mixed and natural convection heat transfer enhancement of nanofluids inside a ventilated square cavity containing different shapes of central cold block

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Abstract :

In the present investigation, the finite volume analysis is used for solving forced, mixed and natural convection of the nanofluid inside a ventilated cavity with a central cold block. The effect of mixed convection is reached by cooling on the inlet port of the vented cavity and uniformly heating on its bottom wall. Water-based nanofluids with Cu, Al\textsubscript{2}O\textsubscript{3}, and TiO\textsubscript{2} nanoparticles at different sizes (25\,nm \leq \,d\,p \leq\, 145\,nm) are chosen for investigation. Dimensionless forms of continuity, Navier Stokes and energy equations with the specified boundary condition have been solved numerically over a wide range of Richardson number (0.1 \leq \,Ri \leq\, 100) and nanoparticle volume fraction (0 \leq \,\varphi \leq\, 0.05). The Grashof number is kept constant at (Gr = 10^4), so that the Reynolds number takes different values (Re = 10, 31.62, 100, and 316.22). The Corcione models have been used to calculate nanofluid effective viscosity and thermal conductivity. The flow patterns, isotherms, and average Nusselt numbers are presented for a constant size of the inlet and exit port (h/H = 0.1). Furthermore, the effects of two different geometric forms of the cold obstacle (square and triangular) on fluid flow and heat transfer rate are also investigated. The results reveal for some cases, that by the reduction of the Richardson number and size of solid particles, the rate of heat transfer is enhanced. It is also noticed that there is an optimal nanoparticles concentration which the maximum mean Nusselt numbers occurs.

Mots clefs: Ventilated square cavity; forced convection; mixed convection; central cold block; nanofluid.
1 Introduction

Forced, mixed and natural convection in ventilated cavities is very important event for many technological processes, such as ventilation of rooms with radiators, cooling of containers, pollution removal, thermal design of buildings, cooling of electronic devices, solar systems and heat exchangers. Recently, enhancement of heat transfers inside the enclosures in several applications needs to introduction of different shapes of the obstacle. Studies on natural convection with obstacles using nanofluid were reported by [1-10]. Work of Kalidasan and Kanna [11] was concentrated on the effect of natural convection inside the open square enclosure containing a central adiabatic square obstacle. Their investigations indicated that the heat transfer on the right and left wall changes with increasing the percentage of nanoparticles concentration. AlAmiri et al. [12] analyzed numerically natural heat transfer convection in square enclosure with a rectangular block placed on the bottom wall. Their investigations showed that increasing the height and width of the rectangular hot block enhances the mean Nusselt number. The same study was performed by Varol et al [13] and Guiet et al [14] for a triangular cavity. El Abdallaoui et al. [15] carried out a numerical simulation of free convection of nanofluid or pure fluid inside a square cavity with decentered and centered triangular heater [16] using the lattice Boltzmann method. They concluded that at high Rayleigh, there is an important effect on heat transfer when the triangular block is placed vertically, but at weak values of Rayleigh the heat transfer is more affected for the decentered position of the block.

Numerous investigations have been found on mixed convection heat transfer in cavities filled with nanofluids [17-21]. The heat transfer by mixed convection inside a vented cavity with an internal heated cylinder was studied numerically by Mamun et al. [22]. They reported that the size of the cylinder and the solid-fluid thermal conductivity has significant effect on thermal field. Mehrizi et al. [23] examined the effect of volume fraction of nanoparticles on mixed convection in a square enclosure having a central hot obstacle with inlet and outlet ports. They concluded that by increasing the solid volume fraction the heat transfer rate is enhanced at different Richardson numbers and outlet port positions. Kalteh et al. [24] presented laminar mixed convection of nanofluid in a lid-driven square enclosure with a triangular heated block. They showed that that increasing the nanoparticles diameter, the volume fraction and Reynolds leads to an increase in mean Nusselt number. Oztop et al. [25] used a lid driven cavity containing adiabatic, isothermal or conductive circular block. They showed that the position and the size of the inner circular body has an important effect on the heat transfer rate. Rahman et al. [26] considered an open enclosure with heated circular cylinder. They concluded that by increasing Reynolds number, the Nusselt number increases. Shahi et al. [27] numerically studied the mixed convection of copper–water nanofluid inside a ventilated cavity. It was found that the average Nusselt number enhances when the volume faction of nanoparticles increased. Bahlaoui et al. [28] used a horizontal ventilated cavity with radiative heated bottom wall having an adiabatic thin partition. They reported that the convective Nusselt number decreases as the radiation effect increases. Laminar mixed convection in a “T” form cavity with heated obstacles submitted to a vertical jet of fresh air from below was investigated numerically by Najam et al. [29].

The main objective of the present study is to examine forced, mixed and natural convection heat transfer inside a ventilated cavity with different shapes of central cold obstacle and with hot temperature at the bottom wall. The first case under investigation is characterized the numerical models used in our study. The computational procedure elaborated in this study is validated against the numerical results of other researches. The effects of water-based nanofluids with Cu, Al₂O₃, and TiO₂ nanoparticles at different sizes (25nm ≤ dp ≤ 145nm) on the heat transfer rate are chosen for investigation. Wide range of parameters such as Richardson number (0.1 ≤ Ri ≤ 100), and volume fraction of nanoparticles (0 ≤ ϕ ≤ 0.05) have been used. The new models of the thermal conductivity and effective
viscosity investigated by Corcione et al. [30] are used to estimate thermophysical proprieties of the nanofluid. Our numerical results are presented in the form of plots of isotherms, streamlines and mean Nusselt numbers to show the influence of nanofluid and design parameters.

2. Problem statement

A schematic view of the studied configurations and coordinate system of the considered ventilated cavity are shown in Fig. 1. It consists of a ventilated cavity having central cold obstacle and uniformly heated with a constant temperature, \( T_h \), from its bottom wall while the other parts of the cavity are all thermally insulated. The cavity is filled with different suspensions of nanoparticles in water. Two cases are considered, i.e., case 1 is a square vented cavity for the height of \( H_w \) where the square cold block are used inside, on the other hand for case 2 we used a triangular block inside where its size is equal to \( (w = 0.2H) \). The physical system is subjected to an external flow of nanofluid which passes through the vented cavity by injection. The nanofluid enters into the vented cavity from the left opening vertical wall which maintained at cold temperature, \( T_c \), and leaves from the right opening vertical one. The length of these openings have a constant value, \( h = 0.1H \). It is assumed that the nanofluid is Newtonian, incompressible and laminar (i.e, \( Ra \leq 10^6 \)) and the base fluid and the nanoparticles are in a thermal equilibrium state. The thermo-physical properties of the nanofluid used in this study are evaluated at the average fluid temperature \( (T_c + T_h)/2 \) as listed in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>( C_p ) (J/Kg K)</th>
<th>( \rho ) (Kg/m³)</th>
<th>( K ) (W/mK)</th>
<th>( \beta \times 10^5 ) (K⁻¹)</th>
<th>( \mu \times 10^6 ) (kg m⁻¹ s⁻¹)</th>
<th>( d_p ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper (Cu)</td>
<td>385</td>
<td>8933</td>
<td>401</td>
<td>1.67</td>
<td>–</td>
<td>25, 85, 145</td>
</tr>
<tr>
<td>( Al_2O_3 )</td>
<td>765</td>
<td>3970</td>
<td>36</td>
<td>0.85</td>
<td>–</td>
<td>25</td>
</tr>
<tr>
<td>( TiO_2 )</td>
<td>710</td>
<td>4157</td>
<td>8.4</td>
<td>0.9</td>
<td>–</td>
<td>25</td>
</tr>
<tr>
<td>Water (H₂O)</td>
<td>4179</td>
<td>997.1</td>
<td>0.613</td>
<td>27.6</td>
<td>855</td>
<td>0.385</td>
</tr>
</tbody>
</table>

3. Mathematical formulation
The governing equations including the two-dimensional transient equations of the continuity, momentum and energy for an incompressible flow are expressed in the following format:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

(2)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}} (T - T_c)
\]

(3)

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

(4)

where the nanofluid effective density, heat capacity, thermal expansion coefficient and thermal diffusivity are calculated from the following equations [31,32]:

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s
\]

(5)

\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s
\]

(6)

\[
(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s
\]

(7)

\[
\alpha_{nf} = k_{nf}/(\rho C_p)_{nf}
\]

(8)

Corcione models [32,33] for the dynamic viscosity and the thermal conductivity of the nanofluid are given by:

\[
\mu_{nf} = \mu_f/ \left( 1 - 34.87 \left( \frac{d_p}{d_f} \right)^{-0.3} \phi^{1.03} \right)
\]

(9)

\[
\frac{k_{nf}}{k_f} = 1 + 4.4Re^0.4Pr^{0.66} \left( \frac{T}{T_f} \right)^{10} \left( \frac{k_p}{k_f} \right)^{0.03} \phi^{0.66}
\]

(10)

\[
Re = \frac{\rho_f u_B d_p}{\mu_f}
\]

(11)

\[
u_B = \frac{2k_B T}{\pi \mu_f d_p^2}
\]

(12)

All terms are defined in the Nomenclature.

The boundary conditions for mixed convection written as:

\[
u = 0, \quad v = 0, \quad T = T_h \quad \text{on bottom wall}
\]

\[
u = 0, \quad v = 0, \quad \partial T/\partial y = 0 \quad \text{on upper wall}
\]

\[
u = 0, \quad v = 0, \quad \partial T/\partial x = 0 \quad \text{on right wall}
\]

\[
u = 0, \quad v = 0, \quad \partial T/\partial x = 0 \quad \text{on left wall}
\]
The following dimensionless variables for mixed convection are defined based on the properties of the pure fluid:

\[ \tau = \frac{t}{H/U_{ref}}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_{ref}}, \quad V = \frac{v}{U_{ref}}, \quad P = \frac{p}{\rho_{nf}U_{ref}^2}, \quad \theta \]

where \( U_{ref} \) is considered to be \( U_0 \) for mixed convection.

Dimensionless numbers for the system are defined as:

\[ Re = \frac{u_{ref}H}{\nu_{nf}}, \quad Ri = \frac{Gr}{Re^2}, \quad Gr = \frac{g \beta_f (T_h - T_c)H^3}{\nu_f^2}, \]

\[ Ra = Gr \cdot Pr = \frac{g \beta_f (T_h - T_c)H^3}{\alpha_f \nu_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \]

The governing equations (1)–(4) are written in the following dimensionless form:

\[ \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{1}{Re \rho_{nf} \mu_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \]

\[ \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{1}{Re \rho_{nf} \mu_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ri \frac{(\rho \beta)_nf}{\rho_{nf} \beta_f} \theta \]

\[ \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re \cdot Pr \alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \]

The dimensionless form of the boundary conditions can be written as:

\[ U = 0, \quad V = 0, \quad \theta = 1 \quad \text{on bottom wall} \]

\[ U = 0, \quad V = 0, \quad \partial \theta / \partial Y = 0 \quad \text{on upper wall} \]

\[ U = 0, \quad V = 0, \quad \partial \theta / \partial X = 0 \quad \text{on right wall} \]

\[ U = 0, \quad V = 0, \quad \partial \theta / \partial X = 0 \quad \text{on left wall} \]

\[ U = 1, \quad V = 0, \quad \theta = 0 \quad \text{on inlet port} \]

\[ \partial U / \partial X = 0, \quad V = 0, \quad \partial \theta / \partial X = 0 \quad \text{on outlet port} \]

\[ U = 0, \quad V = 0, \quad \theta = 0 \quad \text{on block} \]

The dimensionless stream function can be written as:

\[ \psi = - \int_{Y_o}^{Y} U \partial Y + \psi(X,Y_o) \]
The mean Nusselt number of the bottom heated wall is defined as:

$$\overline{Nu} = \frac{1}{H} \int_0^H \frac{k_{nf}(\varphi)}{k} \left\{ \left| \frac{\partial \theta}{\partial Y} \right|_{\text{bottom}} \right\} dX$$  \hspace{1cm} (23)

4. Numerical details

The discretization procedure of the governing equations (Eqs. (17)–(20)) and boundary conditions described by Eq. (21) is based on a finite volume formulation, given by Patankar [39] on a staggered grid. SIMPLE (Semi-Implicit Method for Pressure Linked Equations) is used to solve the coupled pressure–velocity equation while Hybrid Differencing Scheme (HDS) of Spalding [40] is used for the convective terms.

The key step of the finite volume method is the integration of the dimensionless governing equations (17)-(20) over a two-dimensional control volume CV. The integration of the transport equation may be written in a generic form for the variable $\phi$ as:

$$\int_{\text{CV}} \frac{\partial \phi}{\partial t} dV + \int_{\text{CV}} \left( U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} \right) dV = \int_{\Gamma} \Gamma \left( \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} \right) dV + \int_{S} S dV$$  \hspace{1cm} (24)

where $S$ is the source term and $dV=dX.dY$

After integration, the algebraic finite volume equations for the momentum and energy equations are written into the following form:

$$a_P \phi_P = a_W \phi_W + a_E \phi_E + a_S \phi_S + a_N \phi_N + S_{\phi}$$  \hspace{1cm} (25)

where $P$, $W$, $E$, $N$, $S$ denote cell location, west face of the control volume, east face of the control volume, north face of the control volume and south face of the control volume CV respectively (see Fig. 2).

The Line by line application of TDMA (Tri-Diagonal Matrix Algorithm) method [40] is applied on the system of equations until sum of the residuals became less than $10^{-6}$. The developed algorithm was implemented in FORTRAN program.

4.1. Grid independence study
In order to determine a proper grid for the numerical simulation, a square ventilated cavity filled with Cu–water nanofluid ($\varphi = 5\%$) having a square cold obstacle with size $w = 0.2$ is analyzed in two extreme Richardson numbers ($Ri = 0.1$ and 100). The mean Nusselt number obtained using different grid numbers for particular cases is presented in Table 2. As can be observed from the table, a non-uniform $103 \times 103$ grid is sufficiently fine for the numerical calculation.

Table 2
Effect of the grid size on $\overline{Nu}$ for the ventilated cavity filled with the Cu–water nanofluid ($\varphi = 0.05$) having a square cold obstacle with size $W = w/H = 0.2$ for case 1 (see Fig. 1)

<table>
<thead>
<tr>
<th>$Ri$</th>
<th>$63 \times 63$</th>
<th>$83 \times 83$</th>
<th>$103 \times 103$</th>
<th>$123 \times 123$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>23.611</td>
<td>24.071</td>
<td>24.125</td>
<td>24.128</td>
</tr>
<tr>
<td>100</td>
<td>7.009</td>
<td>7.442</td>
<td>7.513</td>
<td>7.515</td>
</tr>
</tbody>
</table>

4.2. Validations

In order to validate the present code against various numerical results available in the literature some different heat convection problems are chosen. The first case is the benchmark problem of natural convection in a square cavity which considered by De Vahl Davis [3] filled with Air ($Pr = 0.71$). Table 3 demonstrates an excellent comparison of the average Nusselt number between the present results and the numerical results found in the literature [34,35]. The numerical results of Iwatsu et al. [36] and Oztop et al. [37] for a top heated moving lid and bottom cooled square cavity filled with air ($Pr = 0.71$). A $100 \times 100$ mesh was used and the computations were done for three different Richardson numbers. Table 4 demonstrates an excellent comparison of the average Nusselt number between the present results and the numerical results found in the literature [36,37] with a maximum discrepancy of about 1.6%. Finally, we chose two other different convection problems (natural and mixed convection) to compare the flow structure. Fig. 3 illustrates the comparison of the isotherms and streamlines with that of mixed convection flow in a lid-driven cavity having a square heater reported by Islam et al. [18] at ($Re=100$, $Ri=0.1$), and with natural convection in a square enclosure containing triangular heater reported by Abdallaoui et al. [15] at ($Ra=10^3$, $Pr=7$).

Table 3
Comparison of $\overline{Nu}$ between the present results and those reported in the literature for a DHC at different $Ra$.

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>$10^3$</th>
<th>$10^4$</th>
<th>$10^5$</th>
<th>$10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Error (%)</td>
<td>0.17</td>
<td>0.13</td>
<td>0.75</td>
<td>0.54</td>
</tr>
<tr>
<td>Barakos and Mitsoulis [34]</td>
<td>1.114</td>
<td>2.245</td>
<td>4.510</td>
<td>8.806</td>
</tr>
<tr>
<td>Relative Error (%)</td>
<td>0.17</td>
<td>0.22</td>
<td>0.55</td>
<td>0.62</td>
</tr>
<tr>
<td>Dixit and Babu [35]</td>
<td>1.118</td>
<td>2.256</td>
<td>4.519</td>
<td>8.817</td>
</tr>
<tr>
<td>Relative Error (%)</td>
<td>0.17</td>
<td>0.71</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Present study</td>
<td><strong>1.116</strong></td>
<td><strong>2.240</strong></td>
<td><strong>4.485</strong></td>
<td><strong>8.751</strong></td>
</tr>
</tbody>
</table>
Grid size   
83²  
83²  
83²  
103²  

Table 4  
Comparison of $\overline{Nu}$ at the hot lid between the present results and those reported in the literature  

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>Average Nusselt number at the hot lid</th>
<th>Iwatsu et al. [36]</th>
<th>Oztop et al. [37]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td></td>
<td>1.34</td>
<td>1.30</td>
<td>1.36</td>
</tr>
<tr>
<td>0.0625</td>
<td></td>
<td>3.62</td>
<td>3.63</td>
<td>3.68</td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td>6.29</td>
<td>6.34</td>
<td>6.29</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of the isotherms and streamlines with that of Abdallaoui et al. [15] ($Ra=10^3$, $Pr=7$) and Islam et al. [18] ($h/H = 0.25$, $Re=100$ and $Ri=0.1$).

5. Results and discussion

In this work, the forced, mixed and natural convection heat transfer inside a ventilated cavity with different shapes of central cold obstacle and with hot temperature at the bottom wall is under study. The influence of the geometric form of cold obstacle (square and triangular), volume fraction ($0 \leq \phi \leq 0.05$), diameter ($25nm \leq d_p \leq 145nm$) and type ($Cu$, $Al_2O_3$ and $TiO_2$) of the nanoparticles on heat transfer rate are discussed for the case 1 and 2 of Fig. 1. The size of each different inner obstacle, $w$, is fixed at 0.2$H$ while the Grashof number, $Gr$, is kept constant at $10^4$. Four different values of Reynolds number ($Re = 10$, 31.62, 100, and 316.22) are chosen to cover forced ($Ri = 0.1$), mixed ($Ri=1$) and natural convection ($Ri\geq10$) regimes. The variation of Reynolds number ($Re = U_0 H/\nu$) is made by changing the velocity ($U_0$) of the incoming flow.

The effects of Richardson number, $Ri$, ranging from 0.1 to 100 and the different shapes of the inner obstacle on streamlines and isotherms are shown in Fig. 4 (a–b). The ventilated square
cavity filled with Cu-water ($\phi = 0.05$ and $d_p = 25\,nm$) is analyzed for case 1 and 2. For comparisons, the streamlines and the isotherms for pure fluid and nanofluid are shown by dashed line and solid line respectively. It can be seen from Fig. 4 that, there is some differences in streamlines and isotherms of pure fluid and nanofluid, which we can explain by the higher viscosity of nanofluid compared to that of the pure fluid which increases the diffusion of momentum in the nanofluid. Generally, the arriving flow strikes to the left side of the cold obstacle, where the flow is directed from the inlet port toward top and down of the cold block and moves to the outlet port. For $Ri = 0.1$ where the forced convection dominates the flow characteristics, the streamlines shown in Fig. 4(a) demonstrate a small rotating cell located in the top right side of the obstacle and one in the bottom left side. The pattern of the streamlines changes with the geometric form of the inner block, the upper rotating cell of the square cold obstacle (case 1) becomes smaller in comparison with that of the triangular block (case 2). This can be explained by the big resistance of the square block, to fluid circulation within the ventilated cavity compared with the triangular block. The corresponding isotherms shown in Fig. 4(b) ($Ri=0.1$) are qualitatively similar to streamlines, indicating the temperature distribution within the ventilated cavity is affected from the forced convection. The isotherms also take the shape of the triangular and square cold obstacle (see case 1 and 2 of Fig. 4(b)). By increasing the Richardson number ($Ri>1$), the main flow traverses bottom of the cold obstacle which is due to decreasing of the incoming flow velocity. At $Ri=1$, mixed convection heat transfer is the dominant regime. The fluid flow occupies the whole of the ventilated cavity and bifurcates near the cold obstacle. The streamlines become almost symmetrical about the line joining the inlet and outlet ports, but a small vortex is developed just at the top of the inlet port, due to increased inertia force. The size of the corresponding vortex is big for the pure fluid in comparison with that of the nanofluid, it can be explained by the role of forced convection which is more significant for the pure fluid. It can be seen from Fig. 4(b) at $Ri=1$, that the isotherms are more tightened with the heated bottom wall in comparison with pervious case ($Ri=0.1$). It is predictable that with increasing the $Ri$ to 10, the vortex which covers the triangular and square cold obstacle due to natural convection increases. Therefore, most flow passes through the bottom region. The corresponding isotherms Fig. 4(b) have more deflection on the left side of the cold obstacle. At high Richardson number ($Ri=100$), the role of natural convection in formation of flow pattern becomes greater and consequently we can see from Fig. 4(a-b) a larger portion of the flow and isotherms get in contact with the heated bottom wall indicating a good convective heat exchange.

The variation of the mean Nusselt number $\overline{Nu}$ of the heated bottom wall for the illustrated cases in Fig. 4 are presented in Fig. 5(a-b). The effects of the Richardson number, size of the solid particles and type of nanoparticles on the mean Nusselt number $\overline{Nu}$ for case 1 and 2 are revealed in Fig. 5.

Fig. 5(a) shows that for the size of solid particles $d_p = 85$ and $145\,nm$ at $Ri=0.1$, where forced convection is dominant, the enhancement of the heat transfer rate is maximum at $\phi = 5\%$. On the other hand, for $d_p = 25\,nm$ there is an optimum volume fraction of nanoparticles which maximize the heat transfer rate, it is about $3\%$. At Richardson number ($Ri = 1$ and $10$), we have seen that for each size of solid particles, the heat transfer rate increases as the volume fraction of nanoparticles ($\phi$) increases so that $\overline{Nu}$ is maximum $\phi = 5\%$. This is due to the fact
that by increasing the volume fraction of the solid particles, the effective thermal conductivity increases and as a result the enhancement of heat transfer rate is maximized at $\varphi = 5\%$. It is worth mentioning that at $\text{Ri}=0.1$, 1 and 10, the heat transfer rate increases with decreasing the nanoparticle diameter and the highest values of the heat transfer rate occur at 25 nm diameter. In addition, it is found that at high Richardson number ($\text{Ri}=100$), where the effects of natural convection are dominant, the size of nanoparticles has an inverse impact on heat transfer rate in comparison with that of $\text{Ri}<100$. The highest values of the heat transfer rate for $\text{Ri}=100$ occur at 145 nm diameter. This is due to the effects of the shape of the cold obstacle (square) where the viscosity effects rise on boundary layer and became with adverse effects on the heat transfer rate. For the corresponding case, enhancement of heat transfer rate is maximized at $\varphi = 1\%$. It is also worth mentioning that by the reduction of the Richardson number, the rate of heat transfer is increased.

![Fig. 4](image-url)

Fig. 4. (a) Streamlines and (b) isotherms inside the ventilated cavity having different shapes of the cold obstacle ($W = w/H = 0.2$), and filled with Cu–water the pure fluid (dashed line) and Cu–water nanofluid (solid line) with $\varphi = 5\%$ and at different $\text{Ri}s$ for cases 1 and 2. $Gr = 10^4$. 
Fig. 5(b) shows that at Ri = 0.1 (i.e. dominant forced convection), the optimum value of φ is between 2 and 3% for different type of nanoparticles. By increasing Richardson number beyond 0.1 (i.e. mixed (Ri =1) and natural convection (Ri> = 10) regimes), the mean Nusselt number increases continuously with the increase of the nanoparticles concentration (φ =5%). Non-significant variation on the heat transfer rate is observed when different kinds of nanofluids are used. It can be seen that among all nanoparticles those nanoparticles with higher thermal conductivity (such as Cu) produce slightly higher rate of heat transfer compared to nanoparticles with lower thermal conductivity (such as TiO2), which indicates that, the mean Nusselt number is not very sensitive to the type of nanoparticles. It is clear from Fig. 5(a-b) that, at all Ri, changing the shape of the cold obstacle from triangular to square, the heat transfer rate increases. From Cases 1 to 2 for φ = 0 , the achieved reduction of $\overline{Nu}$ is approximately 0.09%, 0.22%, 0.18% and 11% at Ri = 0.1,1,10, and 100, respectively. Consequently, the heat transfer rate variation between these two cases is negligible, except at Ri=100 where the shape of the cold obstacle has a significant effect on $\overline{Nu}$.

Fig. 5. Variations $\overline{Nu}$ with respect to the volume fraction of the nanoparticles at different Richardson numbers, size and type of nanoparticles for; (a) case 1 and (b) case 2. $Gr = 10^4$.

6. Conclusion
A computational study is performed to investigate forced, mixed and natural convection heat transfer inside a ventilated cavity with different shapes of central cold obstacle and with a hot
temperature at the bottom wall. Representative results are obtained for wide ranges of the Richardson number \((0.1 \leq Ri \leq 100)\) and the nanoparticle volume fraction \((0 \leq \varphi \leq 0.05)\). Water-based nanofluids with \(Cu, Al_2O_3,\) and \(TiO_2\) nanoparticles at different sizes \((25nm \leq d_p \leq 145nm)\) are chosen for investigation. According to the presented results, the following conclusions are drawn:

- Some differences in the streamlines and the isotherms of pure fluid and nanofluid are observed and this can be explained by the higher viscosity of the nanofluid compared to that of the pure fluid due the higher viscosity of nanofluid which increases the diffusion of momentum.
- By increasing the Richardson number, the main flow direction, changes from top to bottom of the cold block and it has a significant effect on the mean Nusselt number.
- By adding the nanoparticles to the base fluid and increasing the volume fraction of the nanoparticles, the heat transfer rate is enhanced at different Richardson numbers.
- At \(Ri=0.1, 1\) and \(10\) for case 1 (square block), the heat transfer rate increases with decreasing values of the nanoparticle diameter and the highest values of the heat transfer rate occur at 25 nm diameter except, at the high Richardson number \((Ri=100)\), where the size of nanoparticles has an inverse impact on the heat transfer rate in comparison with that of \(Ri<100\) and the highest values of the heat transfer rate occur at 145 nm diameter.
- There is an optimal volume fraction of the nanoparticles for which the maximum heat transfer rate occurs, at low Richardson numbers, \(\varphi_{opt}\) is between 1% and 3% for some cases.
- Among all nanoparticles, those nanoparticles with higher thermal conductivity (such as \(Cu\)) produce slightly higher rates of heat transfer compared to nanoparticles with lower thermal conductivity (such as \(TiO_2\)), which indicates that, the mean Nusselt number is not very sensitive to the type of nanoparticles.
- From Cases 1 (square block) to 2 (triangular block), the achieved reduction of \(\bar{Nu}\) is approximately 0.09%, 0.22%, 0.18% and 11% at \(Ri = 0.1, 1, 10,\) and \(100,\) respectively. Consequently, the heat transfer rate variation between these two cases is negligible, except at \(Ri=100\) where the shape of the cold obstacle has a significant effect on \(\bar{Nu}\).

References


