# Unconditionally stable schemes for gradient flows

Mejdi Azaiez*†1, Chuanju Xu , and Dianming Hou

[1]azaiez (I2M-IPB) – Institut Polytechnque de Bordeaux – 16 Avenue Pey-Berland 33607 Pessac, France

**Résumé**

Many physical problems can be modeled by partial differential equations having the form of gradient flows. The models that come from energy gradient flows describe dynamics driven by a free energy, long used in many fields of science and engineering, particularly in materials science and fluid dynamics references therein.

Typical examples include the Cahn-Hilliard and Allen-Cahn equations for multi-phase flows, for which the evolution PDE system is resulted from the energetic variation of the action functional of the total free energy in different Sobolev spaces.

The energy-based variational framework makes the equations a thermodynamically-consistent and physically attractive approach to model multi-phase flows.

Hence, there is a constant need to develop efficient and accurate numerical schemes for gradient flows models.

While there exist enormous literatures on the investigation for gradient flows models, the investigation of numerical methods for these models are still in high demand.

This is probably due to the fact that the nonlinear terms involved in the free energy functional will yield a severe stability restriction on the time step when implicit/explicit scheme is applied.

In this work, we propose and analyze a new class of schemes based on a variant of the scalar auxiliary variable approaches for gradient flows. Precisely, we construct more robust first and second order unconditionally stable schemes by introducing a new defined auxiliary variable

to deal with nonlinear terms in gradient flows. Our approach consists in splitting the gradient flow into decoupled linear systems with constant coefficients, which can be solved using existing fast solvers for the Poisson equation.

This approach can be regarded as an extension of the SAV method; see, e.g., [Shen et al., J. Comput. Phys. 2018].

The unconditional stability is established, showing that the efficiency of the new approach is less restricted to particular forms of the nonlinear terms.
A series of numerical experiments is carried out to verify the theoretical claims and illustrate the efficiency of our method.

*Intervenant
†Auteur correspondant: mejdi.azaiez@enscbp.fr

**Mots-Clés:** Gradient flows, time, stepping scheme, unconditional stability, spectral method