Influence of the dynamic capillary pressure on the desiccation of homogeneous porous materials

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Abstract :

In the case of a rapid wetting and drying, water movements result in a rapid change in capillary pressure. The models used until now to predict the shrinkage process of most porous media are based on the quasistatic behavior laws and totally neglect the dynamic effects. In this paper, a new model which considering the dynamic effect is put forward to predict the influence of dynamic effects on coupled unsaturated fluid (water-air) transport and mechanical deformation. And 2D geometric model of soil is adopted as homogeneous porous media to verify the meaning and impact of this refined dynamic model through finite element software COMSOL Multiphysics. It proves that dynamic effects cannot be neglected in the procedure of fluid-solid coupling. The faster the relative humidity boundary conditions change, the more obvious the dynamic effect. And the dynamic effect is more significant close to surface.

Key words : Porous mechanics; Dynamic capillary pressure

1 Introduction

Repeated wetting-drying cycles cause significant swelling and shrinkage phenomena which lead to cracks and corrosion of the porous materials. To predict these risks, experimental procedures and highly sophisticated models have been developed by researchers around the world:

- experiments help define probabilistic laws to predict the risk,
- models try to approach real phenomena based on physical relationships.

In the case of wetting-drying cycles, the differences observed between the numerical and experimental results can be explained by a phenomenon little known: the dynamic effect which is proposed by Hassanizadeh and Grey [1] on the capillary pressure (difference between the pressure of the water and that of the interstitial gas in the pores of the porous media). Indeed, in the case of a rapid wetting and drying, water movements result in a rapid change in the capillary pressure. By-also known as capillary pressure is the main cause of delayed deformations (shrinkage) in most of porous media especially the concrete and soil. These deformations are often the cause of the occurrence of micro-cracks and must be mastered. Recent researches have allowed offering capillary pressure law taking into account the dynamic effects [2-5]. However, most of models used until now to predict the drying of porous materials are based on the quasi-static behavior laws and totally neglect the dynamic effects. Few studies have investigated dynamic models.

The main purpose of this paper is to put forward a widespread model based on the poromechanics theory to coupling the deformation of porous matrix and the unsaturation two phases flow (water-air) inside of it under the condition that considering the dynamic capillary effects and phase transformation. Thereafter this widespread model is simplified based on Richards' equation which means that air pressure is constant and equals to atmospheric pressure. And this new model is written into finite element software COMSOL Multiphysics to simulate the water-air two-phase flow inside of soil. The simulation result proves that dynamic effects have a non-negligible influence on the fluid-solid coupling. And its influence is obvious when suffering a sudden and rapid change of surrounding relative humidity. The closer to the surface or interface, the dynamic effect is greater.

2 Governing equation

2.1 Dynamic effects

For the pressure difference $P_G - P_L$ which is the commonly measured quantity, is denoted as P_C^{dyn} and the equilibrium (or 'static') capillary pressure P_C is denoted as P_C^{stat} The difference between dynamic capillary pressure and static capillary pressure is illustrated as :

$$P_C^{dyn} - P_C^{stat} = -\varphi \frac{\partial S}{\partial t} \tag{1}$$

Where φ is the dynamic effects coefficient. Here an empirical equation developed by Stauffer [6] is adopted.

$$\varphi = \frac{\varpi \phi \eta_L}{k\zeta}\left(\frac{P^e}{\rho g}\right)^2 \tag{2}$$

Where ϖ is assumed to be constant and equal to 0.1 for all soils; ϕ is porosity, η_L is viscosity; P^e and ζ are coefficients in the Brooks-Corey formula. P^e is air entry pressure, ζ is pore size distribution index.

2.1 The Deformation of Skeleton

The principle of effective stress is central to pore mechanics. It is the effective stress causes the deformation of the skeleton. The stress–strain equations of a linear, isotropic, elastic porous material under unsaturated flow and infinitesimal transformation conditions are presented here.

$$\sigma_{ij} - bP\delta_{ij} = 2G\varepsilon_{ij} + \lambda \eth \delta_{ij} \tag{3}$$

σ_{ij} is stress tensor; ε_{ij} is strain tensor; b is Biot coefficient; G is shear modulus; λ is Lame's moduli; $\eth$ is the volumetric shrinkage and $\eth = \varepsilon_{ii}$; δ_{ij} is Kronecker delta symbol. Because capillary pressure P_C equals is the difference between gas pressure P_G and liquid pressure P_L . So that

$$P = \left(1 - S_L\right)P_G + S_L P_L = P_G + S_L(-P_C) \tag{4}$$

P is the average pore pressure including the part of water pressure and that of gas mixture pressure; S_L is liquid saturation.

The geometric equation and equilibrium equation are showed as follows

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

$$\sigma_{ij,j} + \rho f_i = 0 \tag{6}$$

u_i is displacement; ρf_i is body forces.

Substituting the geometric equation (5) into constitutive Equation (3) to get an equation to express the stress components σ_{ij} as functions of the derivatives of the displacement components u_i. Replacing those equations into equilibrium equation (6) and neglecting the body force ρf_i . Furthermore considering the dynamic effects it derives the governing equation of deformation:

$$G\nabla \cdot \nabla u + \left(G + \lambda\right)\nabla\left(\nabla \cdot u\right) + b\nabla[P_G + S_L(-P_C) + S_L\varphi \frac{\partial S_L}{\partial t}] = 0 \tag{7}$$

2.2 The unsaturated fluid flow

The mass balance of the liquid water and the gas mixture are:

$$\frac{\partial\left(\phi S_L \rho_L\right)}{\partial t} + \nabla \cdot \left(\rho_L V_L\right) = 0 \tag{8}$$

$$\frac{\partial\left[\phi(1 - S_L)\rho_G\right]}{\partial t} + \nabla \cdot \left(\rho_G V_G\right) = 0 \tag{9}$$

ϕ is Eulerian porosity; ρ is density; V is velocity.

If the variations of the atmospheric pressure are neglected, one can consider the pore air to be essentially at a constant atmospheric pressure. The capillary pressure is now uniquely defined by the water pressure. For convenience it is often assumed that the reference atmospheric pressure $P_{atm} = 0$, so one can write

$$P_C = P_{atm} - P_L = -P_L \tag{10}$$

Because the coupling of fluid flow and the deformation of the skeleton, the velocity of fluid should be rewritten as the Darcy's velocity subtracts the average skeleton velocity $\phi S_L \dfrac{\partial u}{\partial t}$

$$\frac{\partial(\rho_L \phi S_L)}{\partial t} + \nabla \cdot (\rho_L (\frac{k k_{rL}}{\eta_L} \nabla P_C - \phi S_L \frac{\partial u}{\partial t}) = 0 \tag{11}$$

Additionally, the compressibility of the water is neglected here. So that the spatial gradients of the water density and its derivative with time are negligible. The governing equation is rewritten as

$$-\phi \frac{\partial S_L}{\partial P_C} \frac{\partial P_L}{\partial t} + \nabla \cdot \left(-\frac{k k_{rL}}{\eta_L} \nabla P_L \right) = -S_L \frac{\partial \phi}{\partial t} + \nabla \cdot \left(\phi S_L \frac{\partial u}{\partial t} \right) + \nabla \cdot \left[\frac{k k_{rL}}{\eta_L} \nabla \left(\varphi \frac{\partial S}{\partial t} \right) \right] \tag{12}$$

The van Genuchten and the Brooks-Corey formulas that describe the change in $\partial S_L / \partial P_C$, S_L, k_{rL} with the only variable P_L. Until now, the simplified model that governs the coupling between water-air two phases flow and linear elastic porous media under dynamic effects condition has been built.

3 Simulation and conclusion

A 2D geometric model (disc with radius 0.5m) of soil is adopted as homogeneous porous media to verify the meaning and impact of this dynamic model through finite element software COMSOL Multiphysics.

The following Table 1 gives the other necessary variables. Initially, the boundary pressure head is the same as the initial pressure head. While from t=100s to t=200s, the pressure is reduced from -0.1m to -0.2m rapidly, afterward it remains constant.

Table 1. Variables of the simulation

Variables	Unit	Description	Value	Variables	Unit	Description	Value
ρ_L	kg/m^3	Fluid density	1000	θ_s	1	Saturated porosity	0.417
g	m/s^2	Gravity	9.82	θ_r	1	Residual saturation	0.02
K_s	m/s	Saturated hydraulic conductivity	5.8333e-5	H_p	m	Initial water pressure head	-0.1
ζ	1	Brooks-Corey coefficient	0.592	E	Pa	Young's modulus	4e7
H_{ce}	m	Entry pressure head	1/13.8	v	1	Poisson's rate	0.25
l	1	Pore connectivity parameter	1	ρ_s	kg/m^3	Soil density	2000

Figure 1 illustrates the capillary distribution with time. It is obvious that the capillary pressure increases very rapidly in areas close to the boundary. The closer to the boundary, the greater the capillary pressure is. Another important finding is that the capillary dynamic effect is more pronounced at the region close to boundary than the region close to center especially during the initial stage. As time passes, dynamic effects continue to decrease. Figure 2 illustrates the von Mises stress distribution with time. In generally, Dynamic effect causes stress to be slightly lower than that under standard coupling conditions. The stress magnitude is not monotonous along the radius. It undergoes a process of decreasing at first and then increasing when close to the boundary. Figure 3 obviously demonstrate that dynamic coupling causes a leap in stress comparing with standard coupling when the boundary head pressure is rapidly changed in short time. Figure 4 demonstrates von Mises stress under the condition that the boundary pressure head decreases more rapidly from -0.1m to -0.2m within 50s, it seems that the faster the boundary condition changes, the more obvious leap of the von Mises stress. It proves that the stress leap influenced by the change rate of the surrounding environment. A more rapidly change of surrounding relative humidity is easier to cause the larger stress and lead to cracks in porous media.

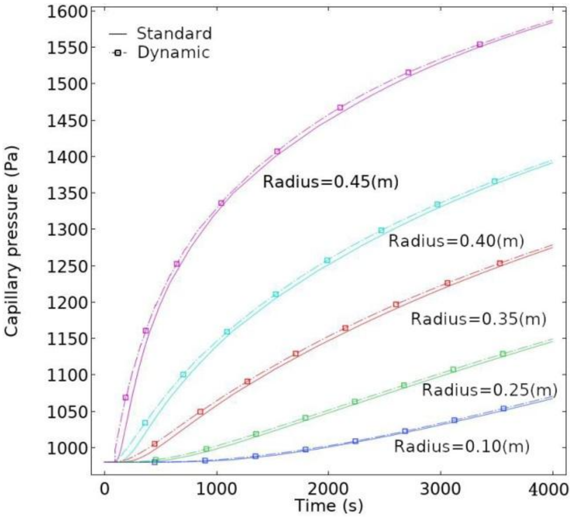

Figure 1. Capillary pressure with Time

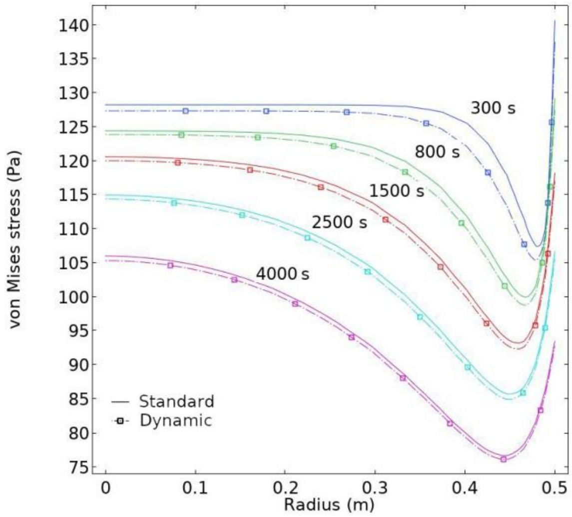

Figure 2. von Mises stress with Radius

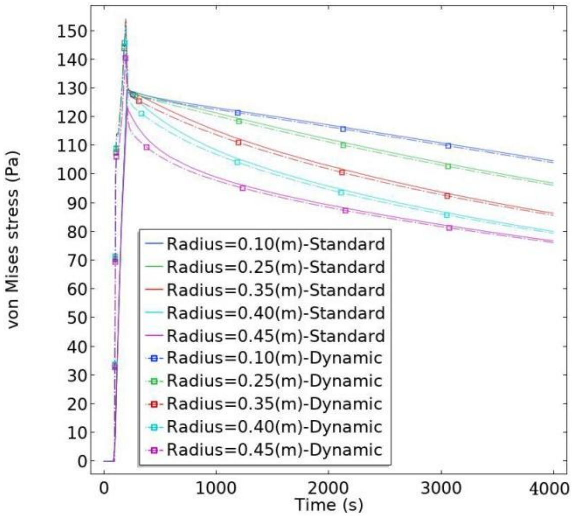

Figure 3. von Mises stress with Time

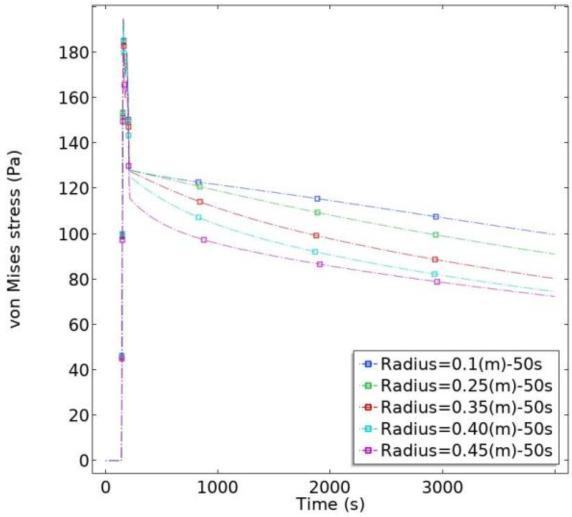

Figure 4. Dynamic von Mises stress with time

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