A macroscopic fatigue criterion for ductile porous material with Drucker-Prager dilatant matrix

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Abstract :

This paper is devoted to determining the shakedown limit states of porous ductile materials with Drucker-Prager matrix under cyclically repeated loads. Using the hollow sphere model and Melan's shakedown theorem based on time-independent residual stress fields, a macroscopic fatigue criterion is derived for the general conditions of cyclic loads. First, the case of the hollow sphere subjected to pure hydrostatic loading is studied and the limit states of collapse by fatigue or by development of mechanism are derived. Then, the general case involving shear effects with any arbitrary cyclic load fluctuations ranging from the pulsating load to the alternating one is considered. The key idea is in two steps : (i) the choice of appropriate trial stress and trial residual stress fields and (ii) then maximizing the size of the load domain in the spirit of the standard lower shakedown theorem. The new macroscopic shakedown criterion depends on the porosity, the friction angle, Poisson's ratio, the two stress invariants of the effective stress tensor and the sign of the third one. Together with the limit analysis-based yield criterion corresponding to the sudden collapse by development of a mechanism at the first cycle, it defines the safety domain of porous materials subjected to cyclic load processes. Interestingly, it is found that the safe domain is little sensitive to variations of the friction angle, however, it is considerably reduced compared to the one under monotonic loads obtained by limit analysis. Finally, a comparative study between the analytical results and numerical predictions performed by micromechanics-based finite element simulations is conducted for different porosities and friction angles.

Mots clefs : Shakedown, Porous material, Drucker-Prager matrix

1 Introduction

The ductile damage of porous materials has aroused a considerable and renewed interest since the pioneering work of Gurson [1] on void growth and overall effective yielding of an ideally plastic hollow sphere and hollow cylinder unit cells. The Gurson-like approach, based on the homogenization theory and the upper bound limit analysis theorem, has been extended to porous solids accounting for material anisotropy [2, 3], void shape effects [4, 5], size effects [6, 7], materials with pressure-sensitive matrix

[8, 9, 10], and non-associated constitutive laws [11]. The dual approach, namely the static limit analysis method, has been recently applied in the strength analysis of porous solids [12, 13]. This approach relies on a suitable choice of statically and plastically admissible trial stress tensors making it more difficult than Gurson's procedure which requires the choice of kinematically admissible trial velocity vectors.

However, structures and mechanical components are often submitted to cyclic mechanical loads and/or temperature variations, acting simultaneously. In fact, an elastic plastic solid subjected to variable repeated actions may fail as a result of alternating plasticity, comprising equal plastic strains of opposite sign leading eventually to local material failure, or by ratcheting (incremental collapse), a progressive accumulation of plastic strains. It may happen also that the structure endures a finite number of cycles with a stabilization of the plastic strains, which is called shakedown. In this case, the response of the structure becomes purely elastic which is beneficial for its strength with respect to high-cycle (or polycyclic) fatigue. The powerful shakedown static and kinematic theorems have been provided for the elastic perfectly plastic materials by Melan [14] and Koiter [15] respectively, and then extended to various and more general constitutive laws [16]. In particular, Melan's theorem, also known as the static shakedown theorem or the lower-bound theorem, follows a static approach with a key concept of an admissible time-independent residual stress field and provides a sufficient condition for shakedown to occur independently on the initial state and the path loading. On the ground of the pioneering Orowan's work [17] on grain plasticity, Dang Van [18] developed his famous shakedown-based approach for the high-cycle fatigue. Generally speaking, the original Dang Van criterion and its improvements assume that the damage occurs at the mesoscopic scale and states that fatigue does not occur if all grains reach a shakedown state.

Although the bibliography on the ductile failure of porous materials under monotonic loads is abundant and renewed, there are few papers dealing with the modeling of ductility under cyclic loadings and most of them concern micromechanics-based numerical approaches. Moreover, all theoretical studies using Gurson-like approach within the framework of limit analysis for the study of voided ductile media subjected to cyclic loads. It is our belief that the natural context for such studies should be the static or kinematic shakedown framework. Accordingly, in a our recent research [19] on shakedown analysis of ductile porous materials with a von Mises matrix under cyclically repeated load, we have proposed a homogenized analytical macroscopic shakedown criterion by considering Gurson's hollow sphere model. Based on a microscopic trial stress field, we have adopted Melan's statical approach to determine the safe limit domain by maximizing the size of the load domain for two limit cyclic loads : (1) the alternating and (2) the pulsating loads. By improving the residual stress field, this approach has been extended to handle general repeated loadings with the definition of the macroscopic stress ratio [20] :

$$-1 \leq R = \Sigma_- / \Sigma_+ < 1 \tag{1}$$

Σ_- and Σ_+ being the minimum and maximum load amplitude during the cyclic loading process, respectively. Thus, alternating load and pulsating one can be considered as two particular loading cases described by $R = -1$ and $R = 0$, respectively. Besides, the load case corresponding to $R = 1$ represents the monotonic load process for which the collapse occurs by development of a mechanism which agreed with numerical simulations.

The aim of this work is to contribute to the theoretical and numerical studies of the effective shakedown of ductile porous materials with an associated Drucker-Prager matrix under cyclic load by the use of Melan's shakedown theorem.

2 Problem formulation

Let us consider an elementary porous cell occupying a bounded volume Ω with a smooth boundary $\partial\Omega$. It is composed of a single void ω embedded in an elastic-perfectly plastic solid matrix $\Omega_M = \Omega - \omega$. The void is bounded by the free-traction surface $\partial\omega$.

Drucker-Prager's yield function of the constitutive law is defined by the convex function F of Cauchy's stress tensor $\boldsymbol{\sigma}$:

$$F(\boldsymbol{\sigma}) = \sigma_e(\boldsymbol{\sigma}) + 3\alpha\sigma_m - \sigma_0 \leq 0 \tag{2}$$

where $\sigma_e = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}}$ is the equivalent stress defined from the deviatoric part $\mathbf{s}$ of the stress tensor $\boldsymbol{\sigma}$, σ_m is the mean stress, σ_0 represents the yield stress, and α is the pressure sensitivity factor related to the friction angle ϕ by :

$$tan\phi = 3\alpha \tag{3}$$

The relationships between the macroscopic stress $\mathbf{\Sigma}$ and macroscopic strain $\mathbf{E}$ fields and their local counterparts $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are obtained by the mean volume operator as follows :

$$\mathbf{\Sigma} = \frac{1}{\mid \Omega \mid} \int_\Omega \boldsymbol{\sigma} \, dV \tag{4}$$

The set of statically admissible stress fields is given by :

$$\mathcal{S}_a = \{\boldsymbol{\sigma} \quad s.t. \quad div\, \boldsymbol{\sigma} = \mathbf{0} \quad in\, \Omega, \quad \boldsymbol{\sigma}\cdot\boldsymbol{n} = \mathbf{0} \quad on\, \partial\omega, \quad \boldsymbol{\sigma} = \mathbf{0} \quad in\, \omega\} \tag{5}$$

where $\boldsymbol{n}$ is the unit outward normal vector of the matrix.

The associated flow law ensures that the plastic strain rate obeys to the normality rule :

$$\dot{\boldsymbol{\varepsilon}}^p = \lambda \frac{\partial F}{\partial \boldsymbol{\sigma}}(\sigma) \,, \tag{6}$$

where $\lambda \geq 0$ is the plastic multiplier.

Let now $(\boldsymbol{\sigma}^E, \boldsymbol{\varepsilon}^E, \boldsymbol{u}^E)$ be the fictitious purely elastic response of the porous shell Ω under the same load. By definition, the residual stress tensor at time t is given by

$$\boldsymbol{\rho} = \boldsymbol{\sigma} - \boldsymbol{\sigma}^E \tag{7}$$

In other words, $\boldsymbol{\rho}$ is the stress field subsisting in the structure after complete elastic unloading of Ω at the instant t. Subsequently, the residual stress tensor is statically admissible with vanishing surface traction on $\partial\Omega$, that is $\boldsymbol{\rho}$ is often called a self-stress field.

For the problem under consideration, $\boldsymbol{\rho}$ belongs to the set of *residual stress fields* :

$$\mathcal{N} = \{\boldsymbol{\rho} \quad | \quad div\, \boldsymbol{\rho} = \mathbf{0} \quad in\, \Omega, \quad \boldsymbol{\rho}\cdot\boldsymbol{n} = \mathbf{0} \quad on\, \partial\omega, \quad \boldsymbol{\rho} = \mathbf{0} \quad in\, \omega\} \tag{8}$$

The key idea of the statical approach is to define the *admissible residual stress fields* (in Melan's sense) $\bar{\boldsymbol{\rho}}(\boldsymbol{x})$, such that [14] :

– $\bar{\boldsymbol{\rho}}$ is time-independent,

– $\bar{\boldsymbol{\rho}}$ is a residual stress field : $\bar{\boldsymbol{\rho}} \in \mathcal{N}$,

– $\bar{\boldsymbol{\rho}}$ is *plastically admissible* in the sense that :

$$\forall \boldsymbol{\sigma}^E \in \mathcal{S}, \quad F(\boldsymbol{\sigma}^E + \bar{\boldsymbol{\rho}}) \leq 0 \quad in \quad \Omega \quad at \quad any \quad time \tag{9}$$

Moreover, if $F(\boldsymbol{\sigma}^E + \bar{\boldsymbol{\rho}}) < 0$ in Ω at any time, $\bar{\boldsymbol{\rho}}(\boldsymbol{x})$ is said to be a *strictly admissible residual stress field*. Hence, the following theorem was proved by Melan [14] :

Melan's theorem : *If a strictly admissible residual stress field $\bar{\boldsymbol{\rho}}$ can be found, the body shakes down.*

3 Macroscopic shakedown citerion under general cyclic loadings

For the general case, it is not possible to obtain the exact solution because of the non linearity of Drucker-Prager yield function. Taking into account the symmetry of the hollow sphere model, the trial stress field is considered as the sum of the two following fields :

– A heterogeneous part inspired from the exact field under pure hydrostatic loadings which is expressed in spherical coordinates :

$$\boldsymbol{\sigma}^{(1)} = \bar{\boldsymbol{\rho}}^{(1)} + \boldsymbol{\sigma}^{E(1)} \tag{10}$$

where the residual stress field in the inner region $a \leq r \leq c$ is inspired from the exact solution [?] in the pure hydrostatic loading :

$$\begin{aligned}\bar{\boldsymbol{\rho}}^{(1)} = A_0 & \left(\left(1 - \left(\frac{a}{r}\right)^{3\gamma} \right) \mathbf{1} + \frac{3}{2}\gamma \left(\frac{a}{r}\right)^{3\gamma} (\boldsymbol{e}_\theta \otimes \boldsymbol{e}_\theta + \boldsymbol{e}_\phi \otimes \boldsymbol{e}_\phi) \right) \\ & - \frac{\Sigma_{m+}}{1-f} \left(\mathbf{1} + \frac{1}{2} \left(\frac{a}{r}\right)^3 (\boldsymbol{e}_\theta \otimes \boldsymbol{e}_\theta + \boldsymbol{e}_\phi \otimes \boldsymbol{e}_\phi - 2\, \boldsymbol{e}_r \otimes \boldsymbol{e}_r) \right)\end{aligned} \tag{11}$$

A_0 being a constant to be determined.
The stress field in the fictitious elastic body is given

$$\boldsymbol{\sigma}^{E(1)} = \frac{\Sigma_m}{1-f} \left(\mathbf{1} + \frac{1}{2} \left(\frac{a}{r}\right)^3 (\mathbf{e}_\theta \otimes \mathbf{e}_\theta + \mathbf{e}_\phi \otimes \mathbf{e}_\phi - 2\, \boldsymbol{e}_r \otimes \boldsymbol{e}_r) \right) \tag{12}$$

– The other part under the pure deviatoric loadings is expressed in spherical coordinates :

$$\boldsymbol{\sigma}^{(2)} = \bar{\boldsymbol{\rho}}^{(2)} + \boldsymbol{\sigma}^{E(2)} \tag{13}$$

where a statically admissible stress field in the fictitious body, deduced from the Papkovich-Neuber solution for the hollow sphere under the pure deviatoric load, was proposed in the previous work [20] in the following form, in the spherical coordinates (r, θ, ϕ) with orthonormal frame $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi\}$:

$$\begin{aligned}\boldsymbol{\sigma}^{E(2)} = -\frac{sign(J_3)\Sigma_e}{3(1-f)} & \left\{ \left[\frac{a^3\left(18a^2 + 5r^2(-5+\nu)\right)(1+3\cos(2\theta))}{2\, r^5(-7+5\nu)} - \frac{1+3\cos(2\theta)}{2} \right] (\mathbf{e}_r \otimes \mathbf{e}_r) \right. \\ & + \left[\frac{a^3\left(27a^2 + 5r^2(1-2\nu) - 3\left(21a^2 + 5r^2(-1+2\nu)\right)\cos^2(\theta)\right)}{2\, r^5(-7+5\nu)} + \frac{-1+3\cos(2\theta)}{2} \right] (\mathbf{e}_\theta \otimes \mathbf{e}_\theta) \\ & + \left[\frac{a^3\left(9a^2 + 25r^2(-1+2\nu) - 45\left(a^2 + r^2(-1+2\nu)\right)\cos^2(\theta)\right)}{2\, r^5(-7+5\nu)} + 1 \right] (\mathbf{e}_\phi \otimes \mathbf{e}_\phi) \\ & \left. + \left[\frac{3a^3\left(12a^2 - 5r^2(1+\nu)\right)\sin(2\theta)}{2\, r^5(-7+5\nu)} + \frac{3\sin(2\theta)}{2} \right] (\mathbf{e}_r \otimes \mathbf{e}_\theta + \mathbf{e}_\theta \otimes \mathbf{e}_r) \right\}\end{aligned} \tag{14}$$

where ν is Poisson's coefficient, Σ_e the macroscopic equivalent stress and J_3 the third invariant of the macroscopic stress deviator.

Noticing that, for the sake of shortness, the full expression of $\bar{\boldsymbol{\rho}}^{(2)}$ is provided in [20].

Consequently, in the matrix Ω_M, the resultant two parameters-based trial stress field in the matrix can be written as :

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} \ , \tag{15}$$

Note that a vanishing stress field is considered in the void ω.

For a variable hydrostatic loading combined with a constant shear loading, we consider the load domain defined by two elementary loads $\boldsymbol{\Sigma}_+$ and $\boldsymbol{\Sigma}_-$, and the axisymmetric macroscopic stress tensor, resulting from (15), takes the form :

$$\boldsymbol{\Sigma}_{\pm} = \Sigma_{m\pm}\mathbf{1} - sign(J_{3\pm})\,\frac{\Sigma_{e\pm}}{3}\left(\mathbf{e}_r \otimes \mathbf{e}_r + \mathbf{e}_\theta \otimes \mathbf{e}_\theta - 2\mathbf{e}_\phi \otimes \mathbf{e}_\phi\right) \tag{16}$$

Considering the yield function (2), the shakedown condition reads :

$$\begin{aligned}&\left[\left(\frac{3\gamma A_0}{2}\left(\frac{a}{r}\right)^{3\gamma} - \frac{3}{2}\frac{\Sigma_{m+}-\Sigma_m}{1-f}\left(\frac{a}{r}\right)^3\right)^2 + \left(\frac{sign(J_{3+})\Sigma_{e+} - sign(J_3)\Sigma_e}{3(1-f)} + K(r)\,A_2\right)^2 P_2(r,\theta)\right.\\ &\left. + \left(3\gamma A_0\left(\frac{a}{r}\right)^{3\gamma} - 3\,\frac{\Sigma_{m+}-\Sigma_m}{1-f}\left(\frac{a}{r}\right)^3\right) P_1(r,\theta)\left(\frac{sign(J_{3+})\Sigma_{e+} - sign(J_3)\Sigma_e}{3(1-f)} + K(r)\,A_2\right)\right]^{1/2}\\ &+ 3\alpha\left[\gamma A_0\left(\frac{a}{r}\right)^{3\gamma} + A_0\left(1 - \left(\frac{a}{r}\right)^{3\gamma}\right) - \frac{\Sigma_{m+}-\Sigma_m}{1-f}\right.\\ &\left. - \frac{5a^3\,(\nu+1)\,\left(3cos^2\theta - 1\right)}{r^3\,(-7+5\nu)}\left(\frac{sign(J_{3+})\Sigma_{e+} - sign(J_3)\Sigma_e}{3(1-f)} + K(r)\,A_1\right)\right] \le \sigma_0\end{aligned} \tag{17}$$

The collapse occurs by fatigue when the yield function vanishes simultaneously for the extreme values of loading, and because $(a/r)^n$ decreases quickly when r increases, the previous condition is satisfied anywhere in the body if it is fulfilled at $r = a$, we obtain :

$$\left[\frac{9}{4}\,(\gamma A_0)^2 + (K(a)\,A_1)^2\,P_2(a,\theta) + 3\,\gamma A_0\,P_1(a,\theta)\,(K(a)\,A_1)\right]^{1/2} + 3\alpha\left[\gamma A_0 - \frac{5\,(\nu+1)\,\left(3cos^2\theta-1\right)}{(-7+5\nu)}\,K(a)\,A_1\right] = \sigma_0 \tag{18}$$

$$\begin{aligned}&\left[\frac{9}{4}\left(\gamma A_0 - \frac{\Delta\Sigma_m}{1-f}\right)^2 + \left(\frac{\Delta(sign(J_3)\Sigma_e)}{3(1-f)} + K(a)\,A_1\right)^2 P_2(a,\theta) + 3\left(\gamma A_0 - \frac{\Delta\Sigma_m}{1-f}\right)P_1(a,\theta)\left(\frac{\Delta(sign(J_3)\Sigma_e)}{3(1-f)} + K(a)\,A_1\right)\right]^{1/2}\\ &+ 3\alpha\left[\gamma A_0 - \frac{\Delta\Sigma_m}{1-f} - \frac{5\,(\nu+1)\,\left(3cos^2\theta-1\right)}{(-7+5\nu)}\left(\frac{\Delta(sign(J_3)\Sigma_e)}{3(1-f)} + K(a)\,A_1\right)\right] = \sigma_0\end{aligned} \tag{19}$$

Due to the linear elastic response when shakedown occurs, one has :

$$\tau = \frac{\gamma A_0}{K(a)\,A_1} = \frac{\gamma A_0 - \frac{\Delta\Sigma_m}{1-f}}{\frac{\Delta(sign(J_3)\Sigma_e)}{3(1-f)} + K(a)\,A_1} = \frac{-\frac{\Delta\Sigma_m}{1-f}}{\frac{\Delta(sign(J_3)\Sigma_e)}{3(1-f)}} \tag{20}$$

Replacing γA_0 and $\gamma A_0 - \frac{\Delta\Sigma_m}{1-f}$, leads to the closed-form macroscopic fatigue criterion :

$$\left\{\begin{aligned}\frac{\Delta\Sigma_m}{\sigma_0} &= -\tau(1-f)\,\frac{2\sqrt{\frac{9}{4}\tau^2 + 3\tau P_1(a,\theta) + P_2(a,\theta)}}{\frac{9}{4}\tau^2 + 3\tau P_1(a,\theta) + P_2(a,\theta) - 9\alpha^2\left(\tau - \frac{5(\nu+1)\left(3cos^2\theta-1\right)}{(-7+5\nu)}\right)^2}\\ \frac{\Delta(sign(J_3)\Sigma_e)}{\sigma_0} &= 3(1-f)\,\frac{2\sqrt{\frac{9}{4}\tau^2 + 3\tau P_1(a,\theta) + P_2(a,\theta)}}{\frac{9}{4}\tau^2 + 3\tau P_1(a,\theta) + P_2(a,\theta) - 9\alpha^2\left(\tau - \frac{5(\nu+1)\left(3cos^2\theta-1\right)}{(-7+5\nu)}\right)^2}\end{aligned}\right. \tag{21}$$

In which two events may occur :

– When $J_{3+} > 0$, the condition is satisfied if it is fulfilled at the equator $\theta = \pi/2$ where the left part of the previous shakedown condition (18) and (19) takes its maximum value, where

$$P_1(a, \frac{\pi}{2}) = \frac{3(5\nu + 5)}{2(5\nu - 7)} \qquad P_2(a, \frac{\pi}{2}) = \frac{225(7\nu^2 - 13\nu + 7)}{(5\nu - 7)^2}$$

– When $J_{3+} < 0$, the condition is satisfied if it is fulfilled at the poles $\theta = 0$ and $\theta = \pi$ where the left part of the previous shakedown condition (18) and (19) takes its maximum value, where

$$P_1(a, 0) = \frac{3(5\nu + 5)}{-5\nu + 7} \qquad P_2(a, 0) = \frac{225(\nu^2 + 2\nu + 1)}{(5\nu - 7)^2}$$

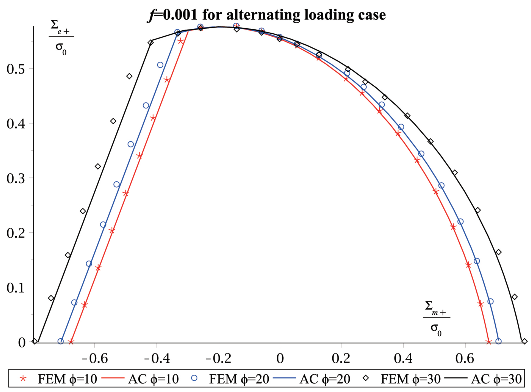
The above macroscopic criterion (21) is established to predict the fatigue limit for hollow sphere with Drucker-Prager matrix in a parametric form which depends on the generalized macroscopic stress triaxiality $\tau = -\frac{3\Delta\Sigma_m}{\Delta sign(J_3)\Sigma_e}$. More precisely, the fatigue limit stress curve can be obtained from this macroscopic criterion for different fixed values of τ.

The set of equations (21) constitutes the main finding in this study.

4 Illustration and assessment of the effective shakedown criterion

The goal of this section is to illustrate the shakedown yield and to validate the accuracy of the analytical results by comparison with numerical ones. To this end, incremental elastic-plastic finite element simulations, which reconstructs in a step-by-step manner the structural response to the applied path loading, are carried out by the use of the software Abaqus Standard [21] by considering a quarter of an axisymmetric model with a spherical void.

The computations are performed for different porosities $f \in \{0.001, 0.01\}$, different friction angles $\phi \in \{10°, 20°, 30°\}$ and with $\sigma_0 = 20$ MPa, $E = 14$ GPa and $\nu = 0.2$. Moreover, the following loading cases are considered for instance : alternating load $R = -1$ and intermediate cyclic load with $R = 1/5$.



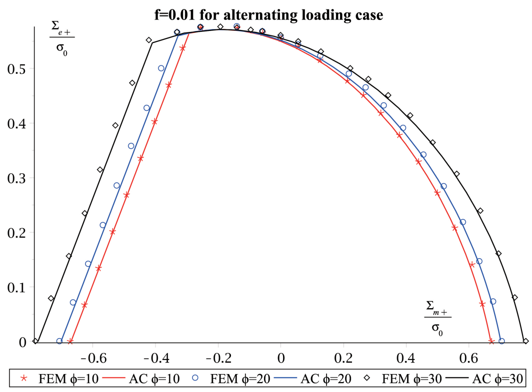


FIGURE 1 – Comparison between the yield surfaces obtained by the analytic criterion and simulations under alternating loadings ($R = -1$) for porosity $f = 0.01$ and $\phi \in \{10°, 20°, 30°\}$.

Fig.1 plotted the macroscopic shakedown domain computed from the established macroscopic fatigue criterion (21) under alternating loading ($R = -1$) for several void volume fractions ($f \in \{0.001, 0.01\}$) and friction angles ($\phi \in \{10°, 20°, 30°\}$). It is worth noting that the collapse by development of a mechanism do not occur in this situation, so the safety domain is only defined by the fatigue criterion. The effective shakedown criterion is found much smaller and completely inside the yield loci obtained under monotonic loads. In the particular case of pure hydrostatic loading, the numerical results fit the exact value $\Delta\Sigma_m^{SD} = \frac{3\,(1-f)}{(3/2+3\alpha)(3/2-3\alpha)}$. This fact is foreseeable since the trial stress field and the residual stress tensor contain the exact solution for the hollow sphere under hydrostatic load.

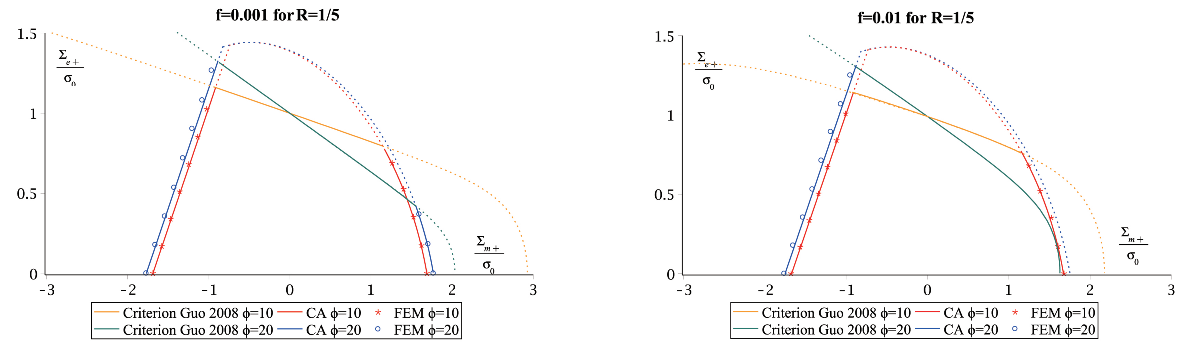


FIGURE 2 – Interaction curve for the intermediate loads with $R = 1/5$ for porosity $f = 0.001$ and 0.01. The analytic safe domain is bounded by solid lines.

Figures 2 displayed the comparison between analytical results and numerical ones of the shakedown limit for $R = 0$ and $1/5$. Unlike the alternating load (for which $R = -1$), the safety domain is obtained at the intersection of the domain defined by the new fatigue criterion and the one proposed in Guo et al. [9], corresponding to the collapse by development of a mechanism at the first cycle. In all figures shown hereafter, **the analytic safe domain is bounded by solid lines**. The second important remark is that the shakedown safe domain is considerably reduced compared to the gauge surface corresponding to the failure under monotonic loading. This ductility reduction is more pronounced in the dominant compression zone ($\Sigma_m < 0$). In addition, these curves confirm that the effect of the friction angle on the safe domain is negligible.

5 Conclusion

In this study we applied Melan's shakedown theorem to derive a homogenized shakedown criterion for ductile porous material with pressure-sensitive dilatant matrix under cyclic repeated loadings, considering the hollow sphere unit cell. The closed-form fatigue criterion in parametric form, depending on the porosity, friction angle, Poisson's ratio and the sign of the third invariant of the macroscopic stress tensor, is able to predict the shakedown limit for all intermediate loading cases ($-1 \leq R < 1$). The safety domain is bounded by this fatigue criterion and by the macroscopic yield strength proposed by Guo et al. [9] corresponding to the collapse by development of a mechanism at the first cycle, inside of which the material is always stable. The established model have been assessed and validated against numerical solutions derived by micomrchanics-based finite element computations by considering a quarter of the hollow sphere for various configurations of porosity and frictions angles. The results provide a good agreement for the general model and, the criterion is strictly conservative to predict the safety domain because of the Melan's statical approach.

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